

## An Experimental Study and Correlation for Differential Settling of Bidisperse Suspensions

P. Krishnamoorthy, I. Reghupathi,\* and T. Murugesan\*\*,+

Department of Chemistry, St. Joseph's College of Engineering, Chennai: 600119, India

\*Department of Chemical Engineering, A. C. College of Technology, Anna University, Chennai: 600025, India

\*\*Chemical Engineering Programme, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750, Tronoh, Perak, Malaysia  
(E-mail: murugesan@petronas.com.my)

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Sedimentation processes have wide practical applications in mineral processing, metallurgical industries, chemical engineering, environmental technologies, water treatment, and bio-process engineering. The sedimentation process is complex in nature since it involves the physical properties of both the solids and the fluids forming the suspensions as well as the hydrodynamic and physico/chemical phenomena that govern particle-fluid and particle-particle behaviour. The aim of this study is to measure experimentally the lower and upper interface velocities in bidisperse suspensions of solids in liquids using liquids and solids with a wide range in physical properties, and more specifically in solids concentration (glass ballotini and sand) covering concentrated and diluted suspensions. The study covers the experimental investigations on the settling rate of bidisperse suspensions in liquids using particles of equal density and at very low particle Reynolds number (Stoke's law region) and the voidage ranges from  $\varepsilon = 0.62$  to 0.95. A correlation is developed for the representation of the experimental data for the particles of two different dimension. The experimental data have been compared with the predictions of the present proposed model as well as the models reported in the literature. An empirical correlation for  $\phi_{S1}$  is also proposed and its dependency on the particle diameter and the concentrations is discussed.

*Key words:*

Settling, suspensions, differential settling, bidisperse suspensions

### Introduction

Evaluation of sedimentation rates in both batch and continuous systems is of theoretical as well as practical importance, wherever there is a need for one discrete phase to move through another continuous phase. Such a need arises when solids are separated from fluids (solids are transported using fluids through pipelines) or when increased interfacial areas are required for efficient mass and heat transfer between the phases.<sup>4</sup> The gravity settling rate of a homogeneous mixture of solid particles in a fluid mainly depends on the size, shape, excess mass of the particles and also the particle fraction. When there are a number of particles dispersed in a fluid, the fall velocity will differ from that of a single particle, due to the mutual interference of the particles (hindered settling). Theoretical and experimental studies showed that even for moderate concentrations, the correction in the fall-velocity becomes significant. For example, for

solids with a volume fraction of  $\phi = 10\%$ , the interparticle distance is about two times the diameter and therefore one might expect hydrodynamic interference, interparticle collision and interaction to be the rule rather than the exception at volume fractions in excess of 2 to 3 percent.<sup>1-3</sup> Owing to the disturbance due to the fluid velocity, the difficulty arises in determining the hydrodynamic interaction between the particles. The falling velocity of the isolated particles decreases to zero with increasing distance and to a lesser extent from the random arrangement of the particles in a real dispersion. The hindrance effect in a mixture of different particles is more complex than the hindrance effect in a monodisperse system. The settling velocity of each component of the mixture is affected to different degrees by the counter flow of the displaced fluid due to the sedimentation of other components. In addition, the surface charges, adsorption and different degrees of aggregation, which are important in a system where physical or chemical flocculation affects the solid-liquid separation.

<sup>+</sup>All correspondence should be addressed to Dr.T. Murugesan  
E mail murugesan@petronas.com.my; tmgesan\_57@yahoo.com.

For the estimation of settling velocities, several models have been reported in the literature.<sup>4–11</sup> These models are mostly empirical rather than mechanistic in structure and therefore are restricted, exclusively for the range of variables investigated by the individual authors. Broadly, they are classified into two categories,

(i) Settling of concentrated suspension [ $\varepsilon_0 < 0.85$ ]<sup>4–7</sup> and

(ii) Settling in very dilute solutions [ $\varepsilon_0 > 0.92$ ].<sup>8–11</sup>

Treatment of systems containing particles of different dimension and/or densities is often based on the concept that each particle settles through a fluid which is modified in density or viscosity by the presence of other settling particles.<sup>4</sup> Alternatively for systems of different particles but of equal density, models based on the concept in which all particles settle against constant interstitial fluid velocity have been proposed.<sup>12</sup> The settling rate of a particle of given size relative to the fluid is calculated from the single particle settling velocity of that particle and the total volume fraction of solids in the suspensions using correlations for mono-disperse suspensions, or an extended model using a correction factor.<sup>5</sup>

*Smith*<sup>13</sup> extended the cell model for the differential settling of particles of a binary mixture by relating the pressure gradients and velocity profiles in the flow of fluid about the particle in the settling systems. Further, the coefficients of the model were evaluated using experimental data. However, the

experimental data showed deviations from the predictions of the model. *Batchelor*<sup>8</sup> proposed a theory for predicting sedimentation velocities in poly-disperse dilute suspensions of spheres at low Reynolds number. The dimensionless sedimentation coefficients appearing in the equations are found to be function of particle diameter ratio, the reduced density ratio, the modified Peclet number based on the relative Stokes velocity and diffusivity, and the inter particle attractive or repulsive forces.

In general, most of the available theoretical models are applicable essentially for highly dilute solutions. Review of literature showed that, till date no unified correlation is available for the prediction of interface velocities in bidisperse suspensions, covering both dilute and concentrated suspensions. Hence, in the present study, it is proposed to develop a generalized correlation covering a wide range in solids concentration, and also to predict the velocities with better accuracy than is presently possible.

## Experimental setup and measurement

The batch sedimentation process of a bidisperse suspension is schematically shown in Fig. 1. A homogeneously dispersed mixture of two particle fractions in a viscous liquid medium, when settled, four different zones were noticed, during the course of sedimentation.

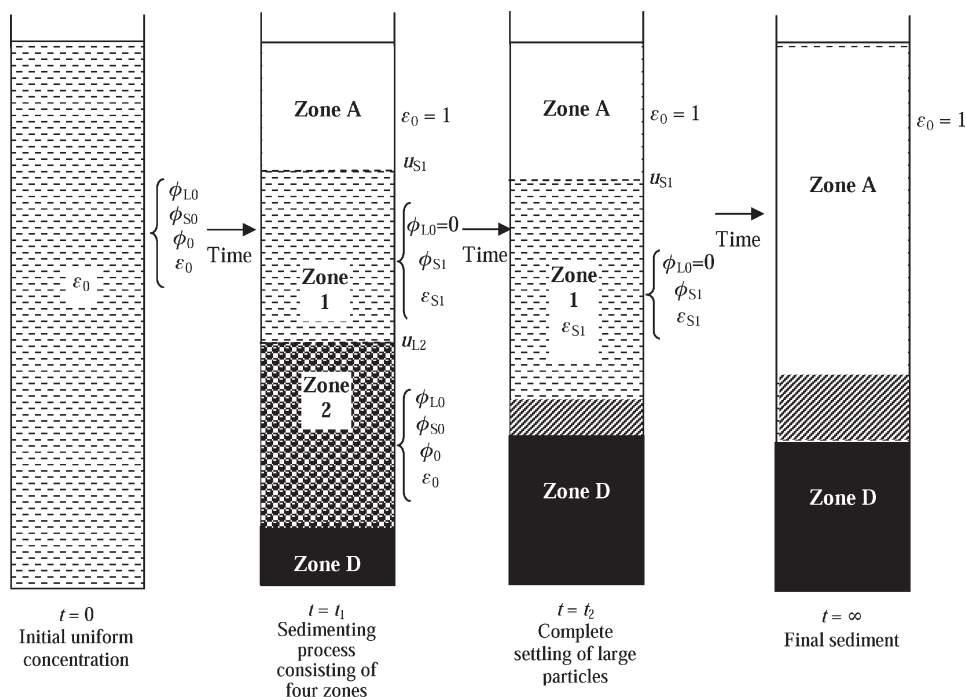


Fig. 1 – Schematics of sedimentation of bidisperse suspension

Initially, at  $t = 0$ , the large and small particles respectively at volume fraction  $\phi_{L0}$  and  $\phi_{S0}$  are uniformly distributed in liquid to form bidisperse suspension with bed voidage  $\varepsilon_0 = (1 - \phi_0) = [1 - (\phi_{L0} + \phi_{S0})]$ . As settling starts, at  $t > 0$ , a zone of clear liquid is seen at the top with  $\varepsilon_0 = 1$ , followed (below) by a uniform suspension of small particles in liquid with  $\varepsilon_{S1}$  ( $= 1 - \phi_{S1}$ ). The interface between clear liquid and the suspension of small particles is denoted 'upper boundary' and its fall speed is recorded 'upper interface velocity,  $u_{S1}$ '. Below the zone of small particles suspension is the zone consisting of large and small particles with solids volume fractions corresponding to the initial value  $\phi_{L0}$  and  $\phi_{S0}$  respectively and the interface between the zones is denoted 'lower boundary' and its fall speed is recorded 'lower interface velocity',  $u_{L2}$ . The upper interface velocity essentially corresponds to the settling of small particles, and the lower interface velocity corresponds to the settling of large particles. Since large particles have higher settling velocity than that of small particles, the fall rate of lower interface is greater than that of upper interface. Due to the downward movement of large particles in zone 2, fluid is displaced upwards elutriating some small particles of zone 2 to zone 1. This gives rise to  $\phi_{S1} > \phi_{S0}$ . With complete settling of large particles, the system has 3 zones and after completion of sedimentation, only two zones remain.

From the top downwards, these consist of: 1) clear liquid (zone A), 2) an upper sedimentation zone where only small particles settle (zone 1), 3) a lower sedimentation zone where both particle species are present as an initially uniform suspension with concentrations equal to their initial concentrations (zone 2) and 4) a sediment layer (zone D). The lower interface velocity ( $u_{L2}$ ) and upper interface velocity  $u_{S1}$  were measured experimentally.

Pycnometric method was employed for the determination of densities of solids as well as liquids (Table 1 & Table 2). Viscosities were determined using Ubbelohde U-tube viscometer. An average value of  $\rho = 2940 \text{ kg m}^{-3}$  was used for Glass ballotini and an average value of  $\rho = 2700 \text{ kg m}^{-3}$  was used for sand, since density variations among these sizes were insignificant. Liquids of widely different in physical properties were used in the present study.

Batch sedimentation experiments were carried out in vertical flat-bottomed glass tubes of different diameters (10 mm, 12 mm and 21 mm i.d.) and 600 mm long held vertically in a metal stand inside a chamber to maintain a constant temperature (303 K). The experiments were conducted in columns of different diameters (10 mm, 12 mm and 21 mm  $D_i$ ) to account for the wall effect adequately. The contents (fluid and the particles of two different sizes

Table 1 – Physical properties of fluids used in this study

Suspending medium	Density at 30 °C $\rho/\text{kg m}^{-3}$	Viscosity at 30 °C $\mu/\text{kg m}^{-1} \text{ s}^{-1}$
diethylene glycol	1132.8	0.2715
ethylene glycol	1113.5	0.0168
glycerol	1298.4	0.5206
80 % glycerol	1227.2	0.0508
70 % glycerol	1280.4	0.0317
60 % glycerol	1204.4	0.0150
sunflower oil	909.9	0.0570
diethanolamine	1165.6	0.1123
80 % diethanolamine	1157.2	0.0678
60 % triethanolamine	1166.8	0.0168
80 % triethanolamine	1187.2	0.0791
5 % poly vinyl alcohol	1074.4	0.0380
7 % poly vinyl alcohol	1078.0	0.1715
transformer oil	936.0	0.0994
sesame oil	976.4	0.0405

Table 2 – Range of variables covered in this study

Variables	This study
solid diameter, $d_p/\mu\text{m}$	
a) glass ballotini	275 – 770 $\mu\text{m}$
b) sand	275 – 655 $\mu\text{m}$
$\rho_p, \text{kg m}^{-3}$	2700, 2940
$\frac{d_L}{d_s}$	1.41 – 2.38
Sphericity	0.83, 1.0
$\phi_{L0}$	
$\phi_{S0}$	0.4 – 4.1
liquid density ( $\rho_f$ ), $\text{kg m}^{-3}$	936 – 1298
liquid viscosity $\mu_f/10^{-2} \text{ kg m}^{-1} \text{ s}^{-1}$	1.5 – 27.15
$\phi_0$	0.05 – 0.38

of glass ballotini and sand) were taken in a graduated glass tube. The downward movement of the lower interface and upper interface velocity was measured by noting the time in seconds at various heights from the top surface of the liquid. The experiment was stopped when the sediment height was about 20 mm below the lower boundary (i.e. zone 2 was about 20 mm in height).

During the experiments, it was observed that the upper region consisted of only small particles and its height increased when the height of zone 2 decreased while the height of the sediments (zone

D) increased as sedimentation progressed. In all the experiments, the lower interface velocity was first measured, followed by the measurement of the upper interface velocity. The experiments for the measurement of the lower interface velocity as well as the upper interface velocity for a given size combination of solids and liquids were repeated for a minimum of 5 to 6 times, and the mean values were recorded. In the present experimental investigation different combinations of each particle sizes were used to have 40 different suspensions, in order to generate data on 171 lower interface velocities and 152 upper interface velocities. The total fraction of solids ranged from 8 % to 34 % by volume.

Apart from the present experimental data, a collection of the experimental data reported in literature (72 data points for lower interface velocity and 64 data points for upper interface velocity) on sedimentation of binary dispersions, were also used for the present analysis.<sup>4,5,10–16,19,20</sup> The reported literature experimental data (Table 3) were used to validate the present model. Table 2 presents the range of the variables covered in the present analysis.

Table 3 – Comparison of RMS deviation of the experimental data with the predictions of the present model and the models reported in literature

	Data points	Present model	Selim et al model	Mirza-Richardson model
Lower interface velocity				
Present data	171	0.06	0.09	0.14
Selim et al data	25	0.05	0.05	*
Mirza-Richardson data	39	0.09	*	0.12
Man Ken Cheung et al. data	8	0.15	*	*
Upper interface velocity				
Present data	152	0.09	0.08	0.13
Selim et al data	25	0.11	*	*
Mirza-Richardson data	39	0.13	*	0.11

\* Data not available in literature

## Model development

The development of the present model is based on the hindered settling functions for monodisperse suspensions. The available monodisperse correlations have been modified suitably to describe the hindered settling in bidisperse systems, taking the properties of the suspensions and inter-particle interactions into consideration. The model proposed

by Mirza and Richardson<sup>5</sup> and later modified by Selim et al.,<sup>4</sup> represents the experimental data better, and makes it applicable over a wider range in experimental conditions, namely both for dilute and concentrated suspensions.

Barnea and Mizrahi,<sup>21</sup> Garside and Al-Dibouni<sup>22</sup> and Davis and Acrivos,<sup>23</sup> presented critical reviews covering both theoretical and experimental research directed at determining hindered settling functions for monodisperse suspensions. The most popular result is the simple formula of Richardson and Zaki<sup>24</sup> relating the settling velocity of the suspension to a fixed horizontal plane, 'u' as a function of solids concentration,  $\phi_0$ , or voidage,  $\epsilon_0$  is

$$\frac{u}{u_t} = \epsilon_0^n = (1 - \phi_0)^n \quad (1)$$

where  $u_t$  is defined as the free settling velocity of the particle. For the Stoke's law region when the ratio of particle to the container diameter ( $d/D$ ), is small, 'n' has a value of about 4.65.

Garside and Al-Dibouni<sup>22</sup> gave the following correlation for the relative velocity between particle and fluid (i.e. slip velocity)

$$\frac{u_s}{u_t} = \epsilon_0^{n-1} = (1 - \phi_0)^{n-1} \quad (2)$$

where 'n' is,

$$\frac{5.09 - n}{n - 2.73} = 0.104 Re^{0.877} \quad (3)$$

and

$$Re = \frac{\rho_f u_t d}{\mu_f} \quad (4)$$

In the above equations,  $u_t$  is the terminal velocity of a single particle settling in the fluid,  $\epsilon_0$  is the bed voidage or porosity,  $u_s$  is the slip velocity and  $Re$  is the particle Reynolds number. Because of the possible influence of the container wall on the settling velocity of the particles the following equations were suggested by Francis,<sup>28</sup> Garside and Al-Dibouni.<sup>22</sup>

The terminal falling velocity of the small particles for the entire range of  $\epsilon_0$  covering dilute and concentrated suspensions and for the large particles ( $\epsilon_0 \geq 0.8$ ), thus limiting to dilute suspensions is estimated in the present model using eq. (5), i.e.

$$u_{t\infty i} = \frac{d_i^2 (\rho_p - \rho_f) g}{18 \mu_f} \quad (5)$$

for  $\begin{cases} \text{large particles, } \epsilon_0 \geq 0.8 \\ \text{small particles} \end{cases}$

To account for particle interactions in the lower sedimenting zone,  $u_{t\infty}$  for the large particles is calculated as if they were settling in a suspension consisting only of the smaller – diameter particles in the Stokes law range.

For the large particles ( $\varepsilon_0 < 0.8$ ), however  $u_{t\infty}$  is calculated by substituting suspension density,  $\rho_s$  instead of  $\rho_f$  in eq. (5):

$$u_{t\infty L} = \frac{d_L^2(\rho_p - \rho_s)g}{18\mu_f} \text{ for } \varepsilon_0 < 0.8 \quad (6)$$

where

$$\rho_s = \frac{(1 - \phi_{l0} - \phi_{s0})\rho_f + \phi_{s0}\rho_p}{(1 - \phi_{L0})} = \frac{\varepsilon_0\rho_f + \phi_{s0}\rho_p}{(1 - \phi_{L0})} \quad (7)$$

$\rho_s$  is the density of a suspension consisting only the small size particles.

For comparison purpose, *Mirza and Richardson*<sup>5</sup> used eq. (5) to estimate  $u_{t\infty}$  and assumed that  $u_{t\infty} = u_t$  for substitution in eq. (1). *Selim et al.*<sup>4</sup> used eq. (6) in combination with eq. (7) to estimate  $u_{t\infty}$  for large particles for the entire range of  $\varepsilon_0$ , and further estimated  $u_t$  using the equations of *Francis*<sup>28</sup> and *Garside and Al-Dibouni*.<sup>22</sup> The range of ' $\varepsilon_0$ ' covered by the above authors was however limited to concentrated suspensions [ $\varepsilon_0 < 0.85$ ] only.

The present model thus assumes that, at higher solids fraction ( $\phi_0 > 0.2$ ), the terminal fall velocities of large particles are influenced by the properties of the fluid as well as the presence of the smaller particles. However, at, low fractions ( $\phi_0 \leq 0.2$ ), the terminal fall velocities of the larger particles are influenced by the properties of the fluids only. The latter observation is a deviation from that of *Selim et al.*<sup>4</sup> who nevertheless limited their experiments and analysis primarily to  $0.20 \leq \phi_0 \leq 0.45$ . *Rong-Her Jean and Liang-Shih Fan*<sup>25</sup> reported that the expression for the buoyancy force should be based on the density of the fluid alone, and not on the density of the suspension. Based on the observations, the suspension density can be used when course particles are falling in a very fine suspension, i.e. a fluid-particle system with a high size ratio. *Doheim et al.*<sup>30</sup> concluded that the *Selim et al.*<sup>4</sup> model deviates substantially in the following two limiting cases: the first one corresponds to suspensions with nearly equal size particles; the second corresponds to the condition when very large particles settle amongst much smaller ones. Further, *Richardson and Meikle*<sup>26</sup> studied the sedimentation of mixtures of ballotini and polystyrene spheres in aqueous alcohol solutions. They observed that at high bed porosity ( $\varepsilon_0 > 0.92$ ), both types of particles were found to settle downward and segregate partially. At low po-

rosity levels ( $\varepsilon_0 < 0.85$ ) polystyrene particles were observed to move upward while ballotini particles settled downwards.

### Lower interface velocity, $u_{L2}$

The slip velocities are defined as

$$u_{SL2} = u_{tL}(1 - \phi_0)^{n_L-1} \text{ for large particles} \quad (8)$$

$$u_{SS2} = u_{tS}(1 - \phi_0)^{n_S-1} \text{ for small particles} \quad (9)$$

where  $n_L$  and  $n_S$  were evaluated using *Garside and Al-Dibouni*<sup>22</sup> eq. (3).

$u_{L2}$  and  $u_{S2}$  were evaluated and expressed in terms of the following equation as suggested by *Selim et al.*<sup>4</sup>

$$u_{L2} = u_{tL}(1 - \phi_0)^{n_L-1}(1 - \phi_{L2}) - u_{tS}(1 - \phi_0)^{n_S-1}\phi_{S2} \quad (10)$$

or,

$$u_{L2} = u_{tL}\varepsilon_0^{n_L-1}(1 - \phi_{L0}) - u_{tS}\varepsilon_0^{n_S-1}\phi_{S0} \quad (11)$$

since,  $\phi_{L2} = \phi_{L0}$ ;  $\phi_{S2} = \phi_{S0}$  and  $\varepsilon_0 = (1 - \phi_0)$

$$u_{S2} = u_{tS}(1 - \phi_0)^{n_S-1}(1 - \phi_{S2}) - u_{tL}(1 - \phi_0)^{n_L-1}\phi_{L2} \quad (12)$$

or,

$$u_{S2} = u_{tS}\varepsilon_0^{n_S-1}(1 - \phi_{S0}) - u_{tL}\varepsilon_0^{n_L-1}\phi_{L0} \quad (13)$$

where  $u_{L2}$  and  $u_{S2}$  refer to the settling velocities of the large and small particles respectively in zone 2, and can be calculated using eq. (10) and (12).  $u_{L2}$  is the lower interface velocity, the rate of fall of the interface between the two sedimentation zones. From the above eq. (10) and (12),  $u_{tL}$  and  $u_{tS}$  were calculated using *Francis*<sup>28</sup> equation by substituting  $u_{t\infty}$  from eq. (5) and (6).

### Upper interface velocity, $u_{S1}$

The fraction of small particles in the upper zone is not directly known but can be calculated through a mass balance. *Man Ken Cheung et al.*<sup>27</sup> measured the interface settling velocities using Nuclear Magnetic Resonance Imaging (NMRI) and reported the concentrations of small particles in upper and lower zones. *Davis and Birdsell*<sup>10</sup> observed that the volume fraction of the smaller particles in the two zones is not equal except in the dilute limit [ $\varepsilon_0 > 0.95$ ].

The volumetric flow rate per unit cross-section at which small particles pass from the lower zone to the upper zone is  $(u_{L2} - u_{S2})\phi_{S2}$ . The rate of increase of volume per unit cross section of the upper zone is  $(u_{L2} - u_{S1})$ , where  $u_{S1}$  is the rate of fall of the upper interface. Thus the concentration of the small particles in the upper zone becomes,

$$\phi_{S1} = \frac{(u_{L2} - u_{S2})\phi_{S2}}{(u_{L2} - u_{S1})} \quad (14)$$

Since the upper zone consists only small particles,

$$u_{S1} = u_{IS}(1 - \phi_{S1})^{nS} \quad (15)$$

Solving the eq. (14) and (15) simultaneously will facilitate the estimation of  $\phi_{S1}$  and  $u_{S1}$ . The condition at which the suspension would settle without segregation of the particles is, when the slip velocities of both sizes are equal. This condition can be met only when  $u_{tL}$  and  $u_{tS}$  are equal. Hence, a binary system consisting of particles of equal densities but different sizes will always segregate to give two settling zones.

## Results and discussions

The experimental data of the present study and the data reported in literature were compared with the predictions of the various available models. The model developed by *Davis and Birdsell*<sup>10</sup> involves sedimentation coefficients, while the model developed by *Smith*<sup>17</sup> contains two functional terms to be evaluated using the experimental data. Both the models are essentially meant for dilute solutions. The predictions using *Man Ken Cheung*<sup>27</sup> model were found to deviate substantially from the experimental data. Thus, a need was felt to extend the present study to modify the Richardson – Zaki<sup>24</sup> equation. Hence, the equation was chosen as the basis for the development of the present model because of its wide applicability and accuracy in predicting the settling rate in monodisperse suspensions.

The model development closely follows to that of *Mirza and Richardson*<sup>5</sup> and *Selim et al.*,<sup>4</sup> which are different from the available models, in accounting for the influence of the presence of small particles on the settling rate of large particles in the lower region. Thus, making it applicable to the settling of both dilute and concentrated suspensions as against the models developed by *Selim et al.*<sup>4</sup> which are applicable only to concentrated suspensions.

Typical experimental data of the present study relating to lower and upper interface velocities are compared in Fig. 2–5 with the predictions of the present model as well as the models reported, which indicate the satisfactory predictions by the present model. The model due to *Mirza and Richardson*<sup>5</sup> predicts higher interface velocities while the model due to *Man Ken Cheung et al.*<sup>27</sup> predicts lower values. The model developed by *Selim et al.*<sup>4</sup> predicts lower interface velocities for dilute suspen-

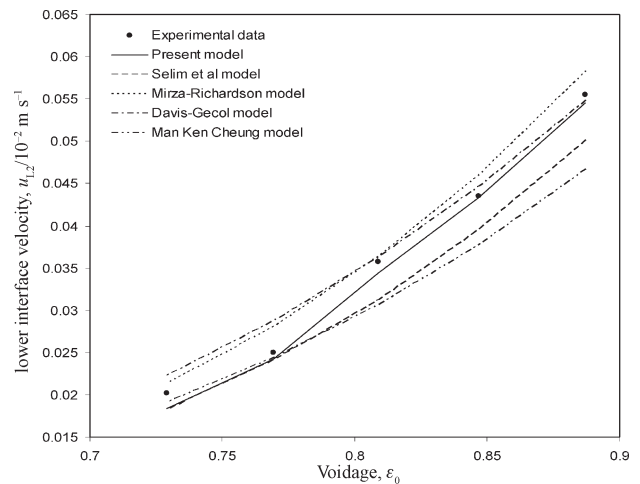


Fig. 2 – Comparison of the present experimental data with the predictions using various models – lower interface velocity

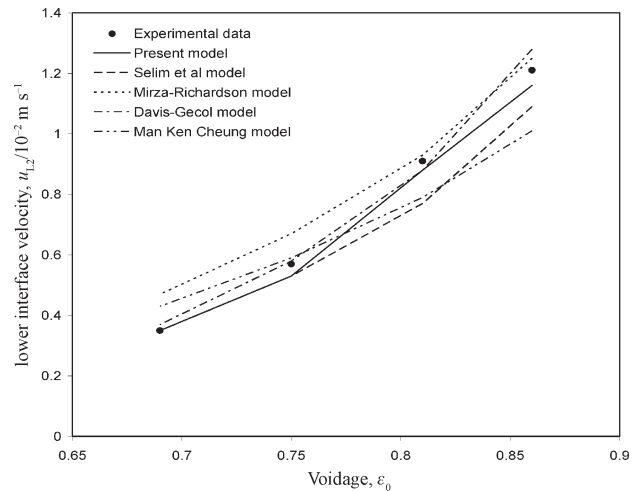


Fig. 3 – Comparison of the present experimental data with the predictions using various models – lower interface velocity

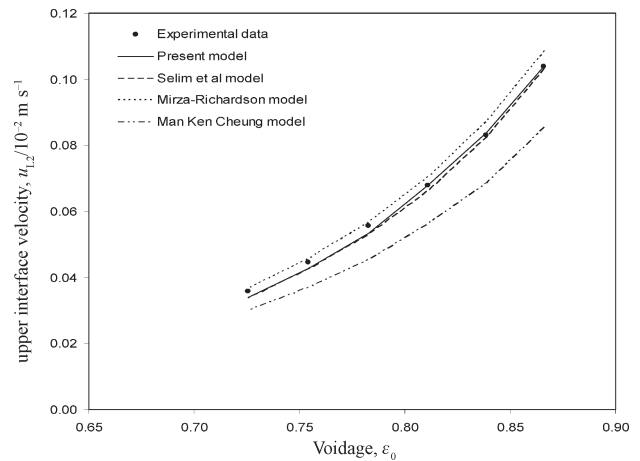


Fig. 4 – Comparison of the present experimental data with the predictions using various models – upper interface velocity

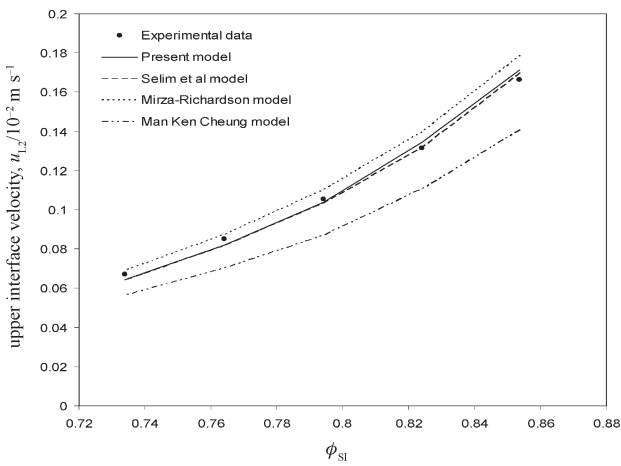


Fig. 5 – Comparison of the present experimental data with the predictions using various models – upper interface velocity

sions while the model developed by *Davis and Gecol*<sup>20</sup> predicts lower velocities, than those predictions using *Mirza-Richardson* model.<sup>5</sup>

In this research, the density difference between the solid particles and fluid medium ( $\rho_p - \rho_f$ ) influences the interface velocities. An increase in ( $\rho_p - \rho_f$ ) increases the lower interface velocity  $u_{L2}$  and

upper interface velocity  $u_{S1}$  (Fig. 9). An increase in fluid viscosity decreases the lower interface velocity  $u_{L2}$  and the upper interface velocity  $U_{S1}$  and also the increase in liquid density decreases the interface velocity and increase in suspension density decreases the slip velocity. The sample data on slip velocity and interface velocity for the different particle and liquid systems is shown in Table 4.

In this research an attempt was also made to study the possible influence of the diameter ratio of large and small particles  $d_L/d_S$  on lower interface velocity. Most of the data obtained in the earlier investigations are limited to  $d_L/d_S > 2$ , while the present study has covered a wide range of  $d_L/d_S$  (1.4 to 2.4). Fig. 6 and Fig. 7 [ $\epsilon_0 < 0.8$ ,  $d_L/d_S \geq 2$  (Fig. 6) and  $\epsilon_0 \geq 0.8$ ,  $d_L/d_S < 2$  (Fig. 7)] compare the present experimental data with the prediction using the present model with an RMS deviation of 0.06. The figures show the influence of diameter ratio, for the range covered in the present study. *Batchelor*<sup>8,9</sup> introduced sedimentation coefficients, which depend upon diameter ratio. He covered a wide range in diameter ratio in theoretical analysis and his study indicated that the influence of diameter ratio on settling velocity is marginal. The range of  $d_L/d_S$  covered in the present study does not explicitly show

Table 4 – Slip velocity and interface velocity for the different particle and liquid systems.

Particle diameter (Glass Ballotini)		Liquid	Voidage $\epsilon_0$	Experiment interface velocity $u/10^{-2} \text{ m s}^{-1}$	Slip velocity $u_s/10^{-2} \text{ m s}^{-1}$
$d_L/\mu\text{m}$	$d_S/\mu\text{m}$				
770	328	glycerol	0.887	0.055	0.057
			0.847	0.043	0.047
			0.809	0.036	0.039
			0.770	0.025	0.029
			0.729	0.020	0.023
770	463	ethylene glycol	0.931	2.423	2.489
			0.909	2.083	2.272
			0.887	1.950	2.071
			0.865	1.818	1.884
770	463	ethylene glycol	0.843	1.538	1.713
			0.822	1.429	1.554
			0.850	0.238	0.281
655	275	80 % diethanolamine	0.824	0.212	0.247
			0.726	0.110	0.120
463	275	60 % triethanolamine	0.839	0.477	0.562
			0.786	0.333	0.381
			0.734	0.232	0.270

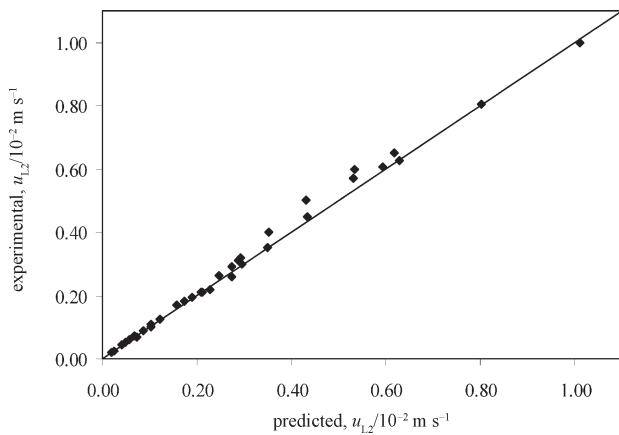


Fig. 6 – Comparison of the present experimental data with the predictions using the present model – effect particle diameter ratio,  $d_L/d_S$  on lower interface velocity

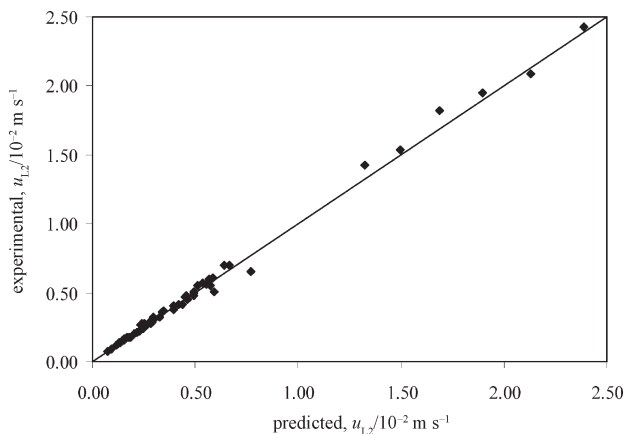


Fig. 7 – Comparison of the present experimental data with the predictions using the present model – effect of particle diameter ratio,  $d_L/d_S$  on lower interface velocity

an influence on the lower interface velocity, in spite of the small particle concentration in the upper region, which is found to be dependent on the ratio. Further, it was observed that the lower and upper interface velocities for sand are lower compared to those for ballotini.

The present research has attempted the influence of particle shape (sphericity) on the lower interface velocity. The lower interface velocity and upper interface velocity for the glass ballotini and sand under identical experimental conditions were compared (Fig. 8).

The lower and upper interface velocities for the glass ballotini ( $\rho_p = 2940 \text{ kg m}^{-3}$ ) are higher than that for sand ( $\rho_p = 2700 \text{ kg m}^{-3}$ ) which is due to higher terminal settling velocity of a single particle in an infinite fluid medium for the ballotini than for sand. Fig. 8 covers all the experimental data obtained using glass ballotini and sand, wherein the data is compared with the predictions using the model for the different ranges in voidage. The pres-

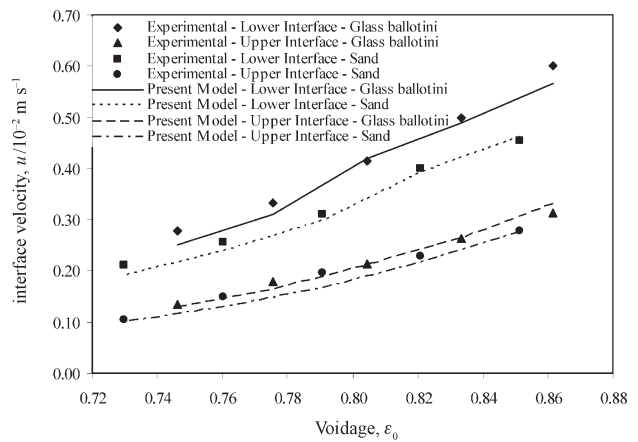


Fig. 8 – Comparison of lower and upper velocity for Glass ballotini and sand

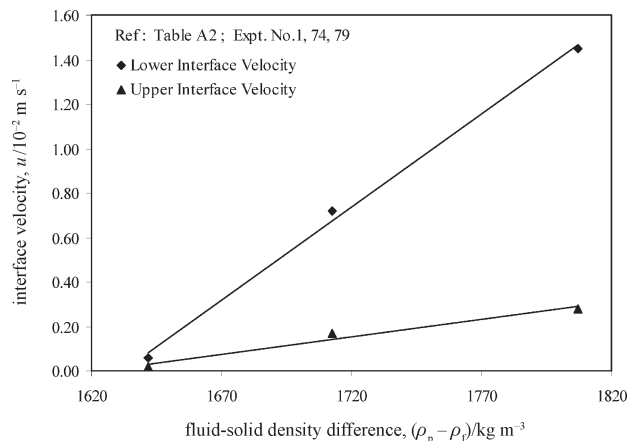


Fig. 9 – Variation of interface velocity with  $(\rho_p - \rho_f)$

ent model, not incorporating sphericity in its formulation is able to match the experimental data satisfactorily, for the range of sphericity covered in the study. This may be due to the reason that the eq. (10) has  $u_{L2}$  and  $u_{tL}$  on either side thus cancelling the effect of sphericity on the ratio of the fall velocities.

In this study an attempt has been made to relate particle (small) fraction in the upper region,  $\phi_{S1}$ , to the fraction of small particle in the lower region which is assumed to be the initial fraction,  $\phi_{S0}$ . *Davis and Birdsell*<sup>10</sup> and *Man Ken Cheung et al.*<sup>27</sup> showed through their experimental measurements that  $\phi_{S1}$  is higher than  $\phi_{S0}$ . The models due to *Mirza and Richardson*<sup>5</sup> and *Selim et al.*<sup>4</sup> as well as the present model formulate elutriation of fine particles from the lower to the upper region by the upflow of the fluid, suggesting  $\phi_{S1} > \phi_{S0}$ .

An analysis of the experimental data of the present study shows that  $\phi_{S1}$  depends on the diameter ratio of particles,  $(d_L/d_S)$  and the large particle fraction,  $\phi_{L0}$  in addition to  $\phi_{S0}$ . Based on the above,  $\phi_{S1}$  is empirically related to  $\phi_{S0}$  as below:



$$\phi_{S1} = 3.25 \left( \frac{d_L}{d_S} \right)^{-0.47} \phi_{L0}^{0.23} \phi_{S0} \quad (16)$$

The  $\phi_{S1}$  obtained from eq. (16) is compared with the predicted  $\phi_{S1}$  using eq. (14) which matches satisfactorily with an RMS deviation of  $\sigma = 0.08$ .  $\phi_{S1}$  estimated from the model equations (based on the experimental data) developed by *Selim* et al.<sup>4</sup> and *Davis* and *Birdsell*<sup>10</sup> with  $\phi_{S1}$  calculated from the empirical eq. (16) were compared and it satisfactorily matches with an RMS deviation of  $\sigma = 0.09$ . Table 3 gives the RMS deviation for the present data and the data reported in the literature when compared with the predictions of the present as well as the available models. Thus, the present model using suspension density for large particles in concentrated suspensions and fluid density for diluted suspensions, predicts  $u_{L2}$  and  $u_{S1}$  with better accuracy. The present data covering the voidage ranges from 0.65 to 0.94 matches with the present model satisfactorily with an RMS deviation of 0.06 for 171 data points of lower interface velocity and 0.09 for 152 data points of upper interface velocity. The available Literature data of *Mirza* and *Richardson*,<sup>5</sup> *Selim* et al.<sup>4</sup> and *Man Ken Cheung* et al.<sup>27</sup> gave satisfactory prediction with present model and the details are given in Table 3.

The present model satisfactorily compares with the experimental data obtained in this study and the data reported in literature covering a very wide range in fluid properties, solid properties and suspension properties of binary dispersion with an RMS deviation of 0.06, which is much lower than the value, reported using the existing models in literature. The present model with no adjustable quantities or arbitrary constants is able to predict the experimental data over a wide range in solids concentration covering both dilute and concentrated suspensions. Though the development of the model equations is similar to the equations proposed by the earlier investigators, the choice of the effective density for the suspension – both  $\rho_s$  and  $\rho_f$  depending upon the solids concentration enhances the applicability of the model with greater accuracy to a wider range in experimental conditions.

## Conclusion

This study, based on a large database, has delineated concentrated suspensions  $\phi_o > 0.2$  from dilute suspensions  $\phi_o \leq 0.2$  during differential settling of bidisperse suspensions. During the settling of large particles through a suspension of large and small particles the influence of the presence of small particles on the settling rate of large particles was considered by taking the suspension density for

the estimation of  $u_{t\infty}$  using Stokes law established for concentrated suspensions. Fluid density was used for dilute suspensions ( $\phi_o \leq 0.2$ ). This differentiation improves the comparison of the experimental data with the predictions of the proposed model. The delineation of suspensions into concentrated and diluted suspensions based on solids concentration finds validity as it satisfactorily correlates the experimental data with the empirical equations. In this study the sphericity is not taken into account due to the reason that eq. (10) has  $u_{L2}$  and  $u_{tL}$  on either side, thus cancelling the effect of sphericity on the ratio of the fall velocities. Usually the small particle fraction in the upper region ( $\phi_{S1}$ ) is calculated by mass balance. There is no empirical correlation available in the literature to calculate  $\phi_{S1}$ . In the present study  $\phi_{S1}$  (eq. (16)) is empirically correlated and compared with the predicted  $\phi_{S1}$  (eq. (14)) which matches satisfactorily with minimum deviation. It is further observed that the influence of size ratio on settling velocity is marginal.

## List of symbols and abbreviations

- $d_p$  – particle diameter, m
- $D$  – column diameter, m
- $h$  – height of sedimentation zones, mm
- $n$  – exponent, equation 2.1
- $Re$  – particle Reynolds number  $\equiv (du_{t\infty} \rho_f / \mu_f)$
- $r$  – radius of particle, m
- $t$  – time from start of sedimentation, s
- $u$  – settling velocity of particle,  $m\ s^{-1}$
- $u_o$  – superficial velocity of fluid,  $m\ s^{-1}$
- $u_S$  – slip velocity,  $m\ s^{-1}$
- $u_t$  – terminal settling velocity of a single particle in a finite fluid medium,  $m\ s^{-1}$
- $u_{t\infty}$  – terminal settling velocity of a single particle in an infinite fluid medium,  $m\ s^{-1}$

## Greek symbols

- $\varepsilon_o$  – porosity or bed voidage, –
- $\phi_o$  – total concentration of particles; volume fraction, –
- $\phi_{L0}$  – total concentration of large particle; volume fraction, –
- $\phi_{S0}$  – total concentration of small particle; volume fraction, –
- $\mu_f$  – viscosity of fluids,  $kg\ m^{-1}\ s^{-1}$
- $\rho$  – density,  $kg\ m^{-3}$
- $\sigma$  – root mean square deviation  $\equiv ###$

## Subscripts

- 1 – zone 1
- 2 – zone 2

- f – fluid  
 L – large particle  
 p – particle  
 S – small particle  
 s – suspension  
 i – large and small particles.

### Abbreviations

RMS – Root Mean Square

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