Identification of Multivariable System Using Relay Tuning

G. S. Dombe and M. Chidambaram*
Department of Chemical Engineering,
Indian Institute of Technology, Madras
Chennai 600 036 India

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In this paper, the recently proposed method by *Ramakrishnan* and *Chidambaram* (2003)[6] on identifying SOPTD transfer function model by asymmetric relay tuning method is extended to identify a multivariable system. The decentralized relay feedback method suggested by *Wang* et. al. (1997)[11] is applied for the $m \times m$ system and m relay tests are required for identifying the entire transfer function matrix. Although, most of the processes can be adequately approximated by a FOPTD model, some of the processes are under-damped and higher order processes can be better incorporated by a SOPTD than a FOPTD model. Certain higher order stable models when approximated to a FOPTD model give a negative time constant, hence, identifying a second order model is necessary. The proposed method is applied to a 2×2 transfer function matrix. The multivariable IMC controllers are designed for the identified model and the closed loop performance of the actual and identified model is compared.

Keywords:
Relay tuning, multivariable system, IMC

Introduction

Aström and *Hägglund* (1984)[1] have suggested an auto-tune procedure, which is very attractive for determining the ultimate gain and ultimate frequency. Here a controller in feedback loop is replaced by an ideal (on-off) relay to generate the sustained oscillations in the output. The critical point (i.e. giving ultimate gain and ultimate frequency) is identified from the limit cycle. The process information at the critical point can be used to identify a suitable transfer function model (*Luyben*, 1987).[5] One of the advantages of this method is that it is easy to control amplitude of the limit cycle by an appropriate choice of the relay amplitude. Besides, it is a closed loop method and works well in highly nonlinear processes. The knowledge of the gain or the time delay should be known for identifying the model using a single relay test. Recently, *Srinivasan* and *Chidambaram* (2003a)[7] have proposed a method to analyze the conventional relay auto-tune data for estimating the three parameters of the FOPTD model using only one relay experiment. They have proposed an additional equation, which along with the phase angle and amplitude criteria gives three parameters. *Srinivasan* and *Chidambaram* (2003b)[8] have also proposed a modified asymmetrical relay feedback method to get improved estimates of the parameters of the FOPTD model. A single relay test is used to evaluate all three parameters of the FOPTD model without any a-priori information about model parameters as time delay or steady state gain. *Ganesh* and *Chidambaram* (2003)[3] extended the method to identify multivariable systems. However, they have identified each subsystem as a FOPTD system. However, for higher order system, identification of FOPTD system may not be adequate and sometimes the identifying the FOPTD model may not be possible.

Recently *Ramakrishnan* and *Chidambaram* (2003)[6] have proposed a method to identify a SOPTD transfer function using the biased relay tuning method. In the present method, the biased relay feedback method proposed by *Ramakrishnan* and *Chidambaram* (2003)[6] for identifying SOPTD model for a scalar system is extended to multivariable system. The extension of auto-tuning technique to multivariable systems is non-trivial and it has attracted much attention in literature. When relay technique is extended to multivariable system using the decentralized relay feedback wherein all the loops are simultaneously subjected to relay feedback. The decentralized relay feed back method is a completely closed loop test. *Luyben* (1987),[4] *Wu* et. al. (1994) have proposed SOPTD identification using independent relay feedback. Also, they have used additional; relay feedback tests to get the extra parameter. *Wang* et al. (1997)[11] have obtained the process steady state gain and frequency response for multivariable system and used it for designing the controller. The objective of this paper is to use *Wang* et al. (1997)[11] method to obtain the steady state gain matrix, and process frequency re-

*E mail: chidam@iitm.ac.in

sponse matrix and use *Srinivasan* and *Chidambaram* (2003)[7,8] method, and *Ramakrishnan* and *Chidambaram* (2003)[6] method to get the model parameters for each element of transfer function matrix.

The paper is organized as follows. In section 2, relay test for multivariable system and estimation of steady state gain matrix and process frequency response is discussed. In section 3, identification of time constant and delay for each element of transfer function matrix is presented. In section 4, comparison of closed loop performance for actual and identified model by designing IMC controller is presented. Simulation examples are given in section 5, followed by conclusion in section 6.

Relay indentification

The Multivariable system is subjected to decentralized relay feedback. Under decentralized relay feedback, the outputs of multivariable system will oscillate with limit cycle after initial transient. The relay feedback procedure as described by *Ganesh* and *Chidambaram* (2003)[3] is performed and the limit cycle data vectors $\boldsymbol{U}_1$ (0), $\boldsymbol{Y}_1$ (0),

$\boldsymbol{U}_1(j\omega)$ and $\boldsymbol{Y}_1$ $(j\omega)$ are found by the equations given by *Ganesh* and *Chidambaram* (2003).[3] Here ω is the frequency of oscillation of the output of the process. *Srinivasan* and *Chidambaram* (2003a)[7a] have proposed method for formulating additional equation for scalar system. This method can be extended to multivariable system to get additional equation.

The vectors $\boldsymbol{U}_1(s)$ and $\boldsymbol{Y}_1(s)$ are given by:

$$\boldsymbol{U}_1(s)=\begin{bmatrix}\int_0^\infty u_1(t)\mathrm{e}^{-st}\mathrm{d}t\\ \int_0^\infty u_2(t)\mathrm{r}^{-st}\mathrm{d}t\end{bmatrix} \quad (1)$$

$$\boldsymbol{Y}_1(s)=\begin{bmatrix}\int_0^\infty y_1(t)\mathrm{e}^{-st}\mathrm{d}t\\ \int_0^\infty y_2(t)\mathrm{r}^{-st}\mathrm{d}t\end{bmatrix} \quad (2)$$

To evaluate the integrals $u(s)=\int_0^\infty u(t)\exp(-st)\mathrm{d}t$ and $y(s)=\int_0^\infty y(t)\exp(-st)\mathrm{d}t$, *Padmasree* and *Chidambaram* (2001) have suggested to use $s_1 = 8/t_s$ where 't_s' is the time at which few (say 3) repeated cycles of oscillations appear in the output. The reason for taking $s_1 = 8/t_s$ is that, for $t > t_s$, because of very small value of the term exp(–st), contribution by subsequent terms is negligible while evaluating the integral value. From the above method Eqs.1 and 2 are solved. Also we have

$$\boldsymbol{Y}_1(0)=\boldsymbol{G}(0)\,\boldsymbol{U}_1(0) \quad (3)$$

$$\boldsymbol{Y}_1(j\omega)=\boldsymbol{G}(j\omega)\,\boldsymbol{U}_1(j\omega) \quad (4)$$

$$\boldsymbol{Y}_1(s_1)=\boldsymbol{G}(s_1)\,\boldsymbol{U}_1(s_1) \quad (5)$$

Since these equations are vector equations, they can not be solved to get $\boldsymbol{G}(0)$, $\boldsymbol{G}(j\omega)$) and $\boldsymbol{G}(s)$. Hence one more ('m' tests for $m \times m$ system) relay feedback test, by slightly varying the relay parameter is performed as described by *Ganesh* and *Chidambaram* (2003).[3] From the first test, vectors of $\boldsymbol{U}_1(0)$, $\boldsymbol{Y}_1(0)$, $\boldsymbol{U}_1(j\omega)$ and $\boldsymbol{Y}_1(j\omega)$ are calculated. Also vectors $\boldsymbol{U}_1(s_1)$ and $\boldsymbol{Y}_1(s_1)$ are calculated from Eqs. (1) and (2). A MATLAB program is written to calculate value of $\boldsymbol{U}(s_1)$ and $\boldsymbol{Y}(s_1)$. This is repeated for all 'm' tests. $\boldsymbol{G}(0)$, $\boldsymbol{G}(j\omega)$ and $\boldsymbol{G}(s_1)$ for 2 × 2 system are calculated as

$$[\boldsymbol{Y}_1(0)\,\boldsymbol{Y}_2(0)]=\boldsymbol{G}(0)[\boldsymbol{U}_1(0)\,\boldsymbol{U}_2(0)] \quad (6)$$

$$[\boldsymbol{Y}_1(j\omega)\,\boldsymbol{Y}_2(j\omega)]=\boldsymbol{G}(j\omega)[\boldsymbol{U}_1(j\omega)\,\boldsymbol{U}_2(j\omega)] \quad (7)$$

$$[\boldsymbol{Y}_1(s_1)\,\boldsymbol{Y}_2(s_1)]=\boldsymbol{G}(s_1)[\boldsymbol{U}_1(s_1)\,\boldsymbol{U}_2(s_1)] \quad (8)$$

Estimation of model parameters

SOPTD model can be expressed as

$$g(s)=\frac{y(s)}{u(s)}=\frac{k_\mathrm{p}\exp(-\theta s)}{(\tau_1 s+1)(\tau_2 s+1)} \quad (9)$$

where, 'k_p' (process gain), 'θ' (time delay), 'τ_1' and 'τ_2' (time constants) are the parameters to be estimated. For this model frequency response can be written by substituting $s = j\omega$ as

$$g(j\omega)=\frac{k_\mathrm{p}\exp(-\theta(j\omega))}{(\tau_1 j\omega+1)(\tau_2 j\omega+1)} \quad (10)$$

From the frequency response matrix $\boldsymbol{G}(j\omega)$, for each element of transfer function matrix is known and is written as

$$g(j\omega)=a+jb \quad (11)$$

Equating the real and imaginary parts of Eqs. (10) and (11) and after simplifications, we get

$$a(1-\tau_1\tau_2\omega^2)-b\omega(\tau_1+\tau_2)-k_\mathrm{p}\cos(\theta\omega)=0 \quad (12)$$

$$b(1-\tau_1\tau_2\omega^2)+a\omega(\tau_1+\tau_2)+k_\mathrm{p}\cos(\theta\omega)=0 \quad (13)$$

Also we have,

$$g(s_1)=\frac{k_p \exp(-\theta s_1)}{(\tau_1 s_1+1)(\tau_2 s_2+1)} \tag{14}$$

The value of k_p for each element is taken from the steady state gain matrix given by Eq. (3). Equations (12), (13), (14) can be solved to get the model parameters τ_1, τ_2 and θ. MATLAB is used to solve Eqs. (12), (13) & (14) to get the model parameters.

Measurement noise is an important issue in the identification problem. Filtering the noise from corrupted signal by using a first order filter is a recommended one (*Aström* and *Hagglund*, 1984).[1] The measurement noise is usually of high frequency while for the controller design the process frequency of interest is usually in the low frequency region. Hence, a low pass filter can be employed to reduce the measurement noise. *Shen* et. al. (1996)[9] have recommended to adopt more oscillation time (periods) in calculating static gain and the critical points of the process.

Comparison of closed loop performance

IMC controller is designed by the method suggested by *Tanttu* and *Lieslehto* (1991)[10] for the identified model to compare the closed loop performance as described by *Ganesh* and *Chidambaram* (2003).[3] Controller ($k_{c,ij}$) for each scalar transfer function $g_{p,ij}$ is first designed. For SOPTD model, PI controller settings are given by

$$(k_p k_c)_{ij}=[2(\tau_1+\tau_2)+\theta)/2\lambda \tag{15a}$$

$$\tau_{I,ij}=(\tau_1+\tau_2)+0.5\theta \tag{15b}$$

where, $\lambda\,[\lambda\geq 0.2(\tau_1+\tau_2)]$ is tuning parameter. The multivariable PI controllers are designed for identified model by the equations given by Ganesh and Chidambaram (2003).[3] Closed loop performance of the actual and the identified model for above designed controller is compared. ISE values are also compared.

Simulation examples

Example 1: The transfer function of a process is given by

$$G(s)=\begin{bmatrix}\dfrac{2e^{-3s}}{1+7s} & \dfrac{e^{-s}}{1+3s}\\[2ex] \dfrac{(s+1)e^{-s}}{(s^2+4s+5)} & \dfrac{e^{-7s}}{1+1.5s}\end{bmatrix} \tag{16}$$

For this process, in the first relay test, relay in loop-1 is a bias with switching levels 1.2 and –1 while, relay in the loop-2 is an ideal relay with switching levels 0.2 and –0.2. The oscillation frequency in both outputs is found to be the same (ω = 0.57). From the process input and output data, the values of $\boldsymbol{U}_1(0)$, $\boldsymbol{Y}_1(0)$, $\boldsymbol{U}_1(j\omega)$ and $\boldsymbol{Y}_1(j\omega)$ are determined. In the second relay test, the switching levels in loop-1 are changed to 1.5 and –1.2. The oscillation frequency in test-2 is nearly same as in test-1. From the process input and output data $\boldsymbol{U}_2(0)$, $\boldsymbol{Y}_2(0)$, $\boldsymbol{U}_2(j\omega)$ and $\boldsymbol{Y}_2(j\omega)$ are determined. From the Equations (7) and (8), the steady state gain matrix and the frequency response matrix are calculated. In both the test, at t_s = 60, all the outputs have few repeated cycles of oscillations. So at $s_1=8/t_s$, $\boldsymbol{U}(s_1)$ and $Y(s_1)$ are also calculated. $\boldsymbol{G}(0)$ and $\boldsymbol{G}(j\omega)$ are calculated as:

$$\boldsymbol{G}(0)=\begin{bmatrix}1.9733 & 1.0739\\ 0.2026 & 0.9979\end{bmatrix} \tag{17}$$

$$\boldsymbol{G}(j\omega)=\begin{bmatrix}-0.479-0.0371j & -0.0567-0.4258j\\ 0.1836-0.1085j & -0.0696+0.7365j\end{bmatrix} \tag{18}$$

In both the test, at t_s = 60, all the outputs have few repeated cycles of oscillations. So, at $s_1 = 8/t_s$ = 0.1333, $\boldsymbol{U}(s_1)$ and $\boldsymbol{Y}(s_1)$ are also calculated from Eqs. (1), (2) and (9). MATLAB is used to calculate $\boldsymbol{U}(s_1)$ and $\boldsymbol{Y}(s_1)$

$$\boldsymbol{U}(0.1333)=\begin{bmatrix}5.5109 & 9.4652\\ -1.3624 & -1.0793\end{bmatrix} \tag{19}$$

$$\boldsymbol{Y}(0.1333)=\begin{bmatrix}2.9732 & 5.8964\\ 0.5387 & 1.3406\end{bmatrix} \tag{20}$$

$$\boldsymbol{G}(0.1333)=\begin{bmatrix}0.6944 & 0.6265\\ 0.1792 & 0.3295\end{bmatrix} \tag{21}$$

Now, if we try to model each element of transfer function matrix as a FOPTD by the procedure given by *Ganesh* and *Chidambarm* (2003),[3] then for $g_{21,}$ we get a negative time constant(τ = –0.1484 & θ = 0.9364). Hence, this transfer function has to be identified by SOPTD model. By solving Eq (8), identified transfer function matrix is obtained as:

$$\boldsymbol{G}(s)=\begin{bmatrix}\dfrac{1.9733e^{-3.05s}}{1+6.98s} & \dfrac{1.07\,e^{-s}}{1+4.02s}\\[2ex] \dfrac{0.6338e^{-0.385s}}{(s^2+1.7411s+3.375)} & \dfrac{0.9979e^{-6.8s}}{1+1.595s}\end{bmatrix} \tag{22}$$

The multivariable PI controller is designed by the *Tanttu* and *Lieslehto*[10] method. For the identified model, the resulting multivariable PI controller is given by

$$\boldsymbol{K}_c = \frac{1}{\lambda}\begin{bmatrix} -15.9051 & 16.3986 \\ 21.1771 & -17.179 \end{bmatrix} \quad (23)$$

$$\boldsymbol{K}_1 = \frac{1}{\lambda}\begin{bmatrix} 0.5638 & -0.5239 \\ -0.1129 & 1.0361 \end{bmatrix} \quad (24)$$

The tuning parameter λ is selected by trial and error method as 60. The performance of actual and identified model is compared. Fig. 1 shows performance of the actual and identified models to be very close to each other. Table 1 gives the ISE values comparison. The ISE value comparison also shows that the identified model is very close to the actual system.

The method proposed in this chapter identifies individual element of transfer function matrix as SOPTD model without any additional tests. Also, it is illustrated that, for some higher order processes when identified as FOPTD model gives negative time constant. So, higher order modeling is necessary for these processes. The proposed method also does not require any a priori knowledge of process.

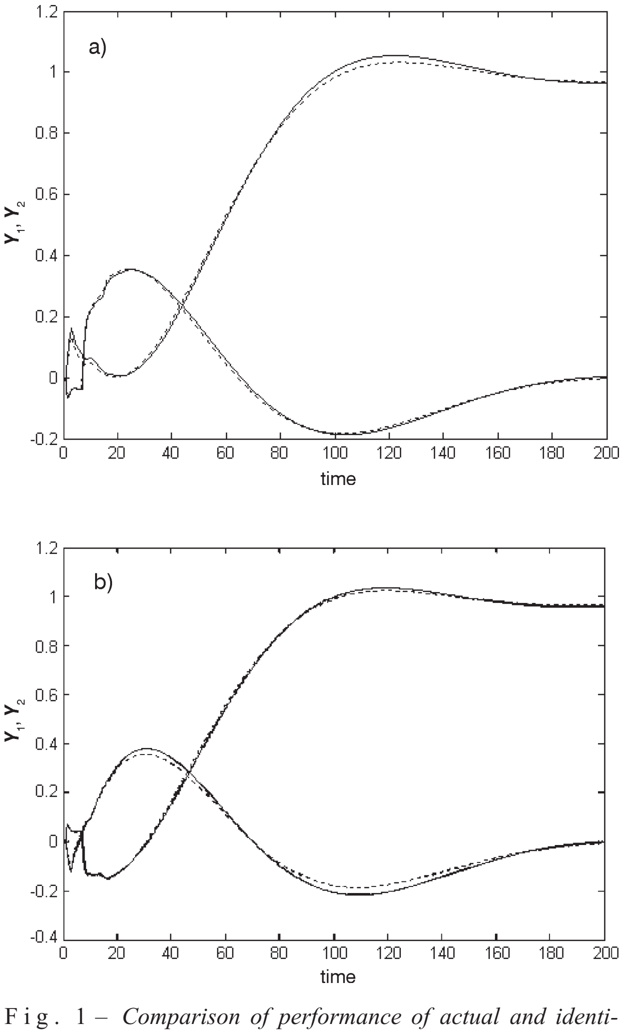


Fig. 1 – *Comparison of performance of actual and identified models for example 1. Upper curves: response; lower curves: interaction; Solid – Actual model. Dot – Identified model: a) Step change in* $\boldsymbol{Y}_1$, *b) Step change in* $\boldsymbol{Y}_2$.

Table 1 – *Closed loop ISE values comparison between the identified model and the actual system for example 1*

Step change in	ISE values for actual model		ISE values for identified model	
	Y_1	Y_2	Y_1	Y_2
Y_1	47.74	4.36	46.68	5.34
Y_2	5	54.38	5.72	54.07

Example 2: The transfer function matrix for the actual system is considered as

$$\boldsymbol{G}(s) = \begin{bmatrix} \dfrac{(2s+1)e^{-10s}}{s^2+2s+1} & \dfrac{e^{-0.5s}}{s^2+3s+4} \\ \dfrac{(s+1)e^{-s}}{s^2+4s+5} & \dfrac{(s+2)e^{-8s}}{s^2+2s+2} \end{bmatrix} \quad (25)$$

For this process, in the first relay test, relay in loop-1 is bias one with switching levels 1.5 and –1 while, relay in the loop 2 is an ideal relay with switching levels 0.2 and –0.2. The oscillation frequency in both outputs is found to be the same (ω = 0.304). From the process input and output data $\boldsymbol{U}_1(0)$, $\boldsymbol{Y}_1(0)$, $\boldsymbol{U}_1(j\omega)$ and $\boldsymbol{Y}_1(j\omega)$ are determined. In the second relay test. The switching levels in loop-1 are changed to 1.8 and –1.2. The oscillation frequency in test-2 is nearly the same as in test-1. From the process input and output data $\boldsymbol{U}_2(0)$, $\boldsymbol{Y}_2(0)$, $\boldsymbol{U}_2(j\omega)$ and $\boldsymbol{Y}_2(j\omega)$ are determined. From Eqs. (7) and (8) steady state gain matrix and frequency response matrix are calculated in both the test, at $t_s = 60$, all the outputs have few repeated cycles of oscillations. Using $s_1 = 8/t_s$, $\boldsymbol{U}(s_1)$ and $\boldsymbol{Y}(s_1)$ are calculated. We get $\boldsymbol{G}(0)$ and $\boldsymbol{G}(j\omega)$ as:

$$\boldsymbol{G}(0) = \begin{bmatrix} 0.9993 & 0.2383 \\ 0.2 & 1.0015 \end{bmatrix} \quad (26)$$

$$\boldsymbol{G}(j\omega) = \begin{bmatrix} -1.0673 - 0.0587j & 0.2404 - 0.0658j \\ 0.2 - 0.052j & 0.8608 - 0.5336j \end{bmatrix} \quad (27)$$

In both the tests, at $t_s = 60$, all the outputs have few repeated cycles of oscillations. So at $s_1 = 8/t_s = 0.1333$, $\boldsymbol{U}(s_1)$ and $\boldsymbol{Y}(s_1)$ are also calculated from Eqs. (1), (2) and (9). We get the $\boldsymbol{G}(0.1333)$ as:

$$\boldsymbol{G}(0) = \begin{bmatrix} 0.2598 & 0.21183 \\ 0.1786 & 0.3211 \end{bmatrix} \quad (28)$$

The identified transfer function matrix is given by:

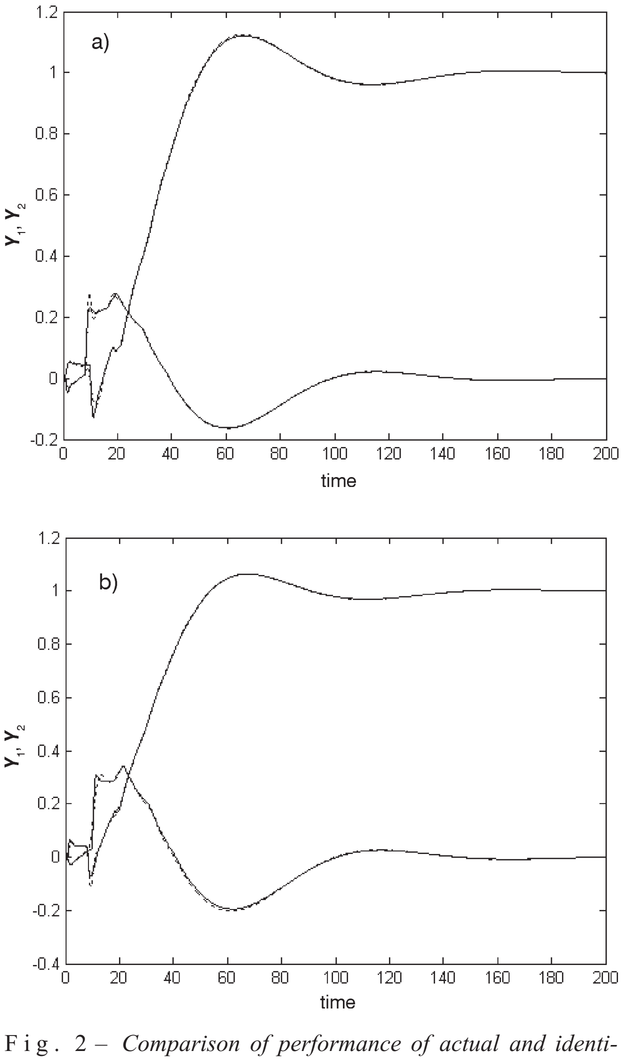
$$\boldsymbol{G}(s) = \begin{bmatrix} \dfrac{e^{-8.76s}}{1.589s^2+1.26s+1} & \dfrac{(0.2386)e^{-0.08s}}{0.766s^2+0.7572s+1} \\ \dfrac{(0.2)e^{-0.086s}}{0.6199s^2+0.7205s+1} & \dfrac{e^{-8.1s}}{0.2154s^2+0.394s+1} \end{bmatrix} \quad (29)$$

The multivariable PI controller is designed based on the identified model as:

$$\boldsymbol{K}_{\mathrm{c}} = \frac{1}{\lambda}\begin{bmatrix} -5.8396 & 7.7671 \\ 6.7989 & -4.5989 \end{bmatrix} \tag{30}$$

$$\boldsymbol{K}_1 = \begin{bmatrix} 1.0504 & -0.2506 \\ -0.2111 & 1.0504 \end{bmatrix} \tag{31}$$

The tuning parameter (λ) is selected by trial and error as 30. The closed loop responses are obtained by using SIMULINK package for a set point change in the set point of y_1 first and separately in y_2. Both, the model and the actual system responses match very well. The ISE values are calculated and given in Table 2. Very closed match is obtained between the performance of the same controller on the process and on the model.



F i g . 2 – *Comparison of performance of actual and identified models for example 2 upper curves: response; lower curves: interaction; Solid – Actual model. Dot- Identified model: a) Step change in* $\boldsymbol{Y}_1$*, b) Step change in* $\boldsymbol{Y}_2$.

T a b l e 2 – *The closed loop ISE values comparison for the actual and identified models for example 2*

Step change in	ISE values for actual model		ISE values for identified model	
	Y_1	Y_2	Y_1	Y_2
Y_1	27.107	1.8417	27.18	1.86
Y_2	2.77	24.004	2.81	23.99

Conclusions

Asymmetric relay feedback method for scalar system (*Ramakrishanan* and *Chidambaram*; 2003) is extended to a multivariable system. Decentralized relay feedback tests are carried out. The method identifies each element of the transfer function matrix as either FOPTD or SOPTD model without any a priori knowledge of model parameters such as time delay or steady state gain. It is shown that for some higher order processes when identified as FOPTD model gives a negative time constant. Modeling the system as a SOPTD gives a better performance. Simulation results on two examples show that the present method gives identified model giving a similar closed performance as that of the actual system with the same controllers.

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