MINLP Optimization of Plate fin Heat Exchangers

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Plate Fin Heat Exchanger design is a very complex task. In most cases, heuristic-based procedures are used. In order to improve the company profits, the PFHE design problem is stated according to mathematical programming techniques. First of all, objective functions such as manufacturing cost, physical volume are detailed as well as operating and manufacturing constraints. Finally, optimization variables including the geometrical fin parameters are described. Since most of the geometrical parameters of the exchanger (core number, geometrical fin parameters, etc …) have discrete values, this formulation results in a Mixed Integer Non Linear Programming (MINLP) problem. Different solution strategies are discussed. For example, the solution of the relaxed problem using a Successive Quadratic Programming (SQP) algorithm. Another example is the solution of the original MINLP problem using Simulated Annealing (SA) or Branch and Bound (BB) algorithms. The efficiency of the developed tool is illustrated by two industrial case studies : the manufacturing cost reduction is greater than 10 %.

Keywords:
Plate fin heat exchanger; mixed integer non linear programming, simulated annealing

Introduction

Computer Aided Process Engineering (CAPE) tools are now widely used in large industrial groups. This is especially true for simulation tools. Concerning optimization tools, the gap between academic research and that done in medium size companies is still great. This paper deals with the introduction of MINLP optimization techniques in the design procedure of a Plate Fin Heat Exchanger (PFHE) manufacturer NORDON CRYOGENIE.

Brazed aluminum Plate Fin compact Heat Exchangers are widely used for cryogenic and aeronautical applications. PFHE is described in figure 1. Parting sheets and fins are stacked alternatively. Fins are used in each layer in order to ensure the effective distribution of fluids and to improve thermal effectiveness. A key point of PFHE design is that good thermal efficiency generally obliges a consequent pressure drop. Serrated fins are more appropriate for high thermal efficiency. Perforated fins are used when diphasic flows are involved. The single core distributors allow the distribution of the fluids into the different layers. The complete exchanger is an assembly of single cores.

Such exchangers may involve up to 18 different fluids. For the considered applications, aluminum is used because of its efficient mechanical properties and its lightness (from 800 to 1200 kg/m^3). The maximum operating pressures is equal to 8.10^3 kPa. Temperatures vary from –269 °C to +56 °C. Thanks to its large area of exchange (1500 m^2/m^3), very slight temperature differences (between hot and cold streams) are feasible. The main drawback is that fouling may irreversibly damage the brazed exchanger.

Since PFHE design is of great industrial interest, there are many recent works that have been presented. Zhu and Pua[1] proposed new methodology for the optimization of fin selection in the context of the overall design problem. The objective is to help engineers in designing the overall network using compact heat exchangers which have been proved to be more efficient than shell-and-tube ones. Jia *et al*[2] proposed a software package for the optimization and the drawing of compact heat exchangers. The NLP optimization problem is solved by using non derivative techniques: the Complex and the Rosenbrock method. Non convexity is handled starting from numerous initial points. Wang and Sundén[3] developed an approach to the design of plate heat exchangers where the full utilization of allowable pressure drops is the objective of the design. This work is dedicated to chevron-type plate heat exchangers.

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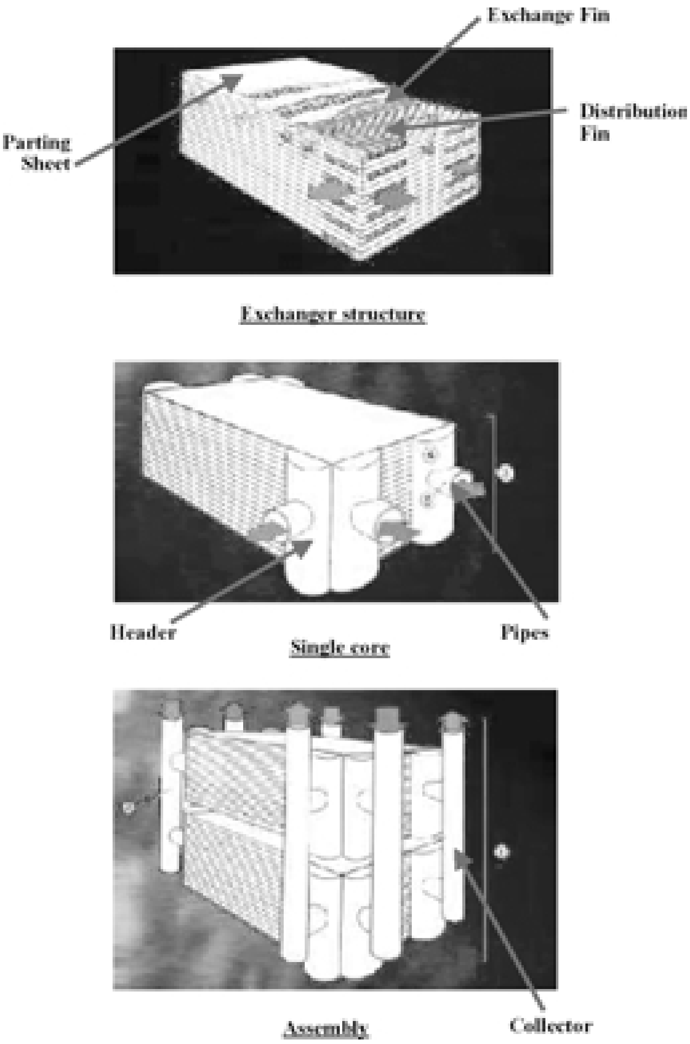

Fig. 1 – *Plate fin heat exchanger description*

NORDON CRYOGENIE has developed a heuristic-based tool for PFHE modeling and design: COLETH. Using such a program it is possible to achieve a fairly accurate approximation of the optimal design. During this classical design procedure, the designer's experience and know-how are of critical importance. Those points are discussed in section 2.

In order to improve company profits, development of efficient and accurate tools for optimal design of PFHE has become a priority in most companies. Our main goal is to integrate mathematical programming techniques in the COLETH program in order to achieve the optimal design of PFHE. In previous works[4], we have considered the relaxed formulation and solution of the PFHE design problem. Geometrical parameters were considered as continuous optimization variables. The resulting relaxed NLP optimization problem was solved using a Reduced Hessian Successive Quadratic Programming (SQP) algorithm.[5] Actually, most of the geometrical parameters, from a manufacturing point of view, have discrete values. In the present work, the original Mixed Integer Non Linear Programming (MINLP) problem is considered. Both Branch and Bound (BB) as well as Simulated Annealing (SA) algorithms are used. The formulation and solution of the optimization problem are considered in sections 3 and 4.

In the last section, two industrial examples are illustrated. The tool which is developed is proven to be efficient. Convergence properties of the different algorithms are discussed. General trends for PFHE design are pointed out.

Classical design procedure

Introduction

The general design problem to be solved by the NORDON company engineers can be stated as follows. Inlet and outlet temperatures, enthalpies, transport properties and flow rates are supposed to be known. Thus the total duty Q^{Tot} is fixed. Maximum pressure drops are also specified for each stream. Geometrical parameters of the PFHE have to be calculated. In order to solve this problem, NORDON Cryogenie has developed a tool: COLETH.

Definitions

COLETH is an heuristic-based procedure for PFHE design. Two modes are available: the automatic sizing mode and the sizing mode. In both cases, the user must supply the following data: flow rates, inlet and outlet temperatures, enthalpies, transport properties and maximum pressure drops. It should be pointed out that the COLETH program is coupled with the PROPHY database. The PROPHY database includes pure component data banks and a thermodynamic model library. For cryogenic applications, the model developed by Benedict, Webb, Rubbin and Starling (BWRS) is of particular interest. Given this, one can use the interface with the PROPHY database in order to generate the required enthalpies and transport properties.

In the **automatic sizing mode**, all the geometrical parameters of the heat exchanger (parting sheet thickness, bar width, number of cores, core width, fin geometry, number of layers, distributor geometry, length of the core …) as well as pressure drops and the cost of the heat exchanger are computed.

In the **sizing mode**, geometrical parameters of the heat exchanger (except core length) are supplied by the user. The procedure then computes the pressure drops, the cost of the heat exchanger and the necessary length of the core.

A **section** is the space between any stream inlet or stream outlet. In most cases, more than 60 % of the total duty is exchanged in one section: the Main Duty Section. As discussed here after, this section is of particular interest during the optimization procedure.

Since PFHE can involve several hot and cold streams, the composite curve is built: temperature versus duty for both hot and cold streams. The total duty is divided into **intervals** of equal length. In each duty interval, the physical properties of the streams (specific heat, dynamic viscosity…) are assumed to be constant. The selection of the number of intervals is discussed in section 2, "Main steps of the COLETH design procedure" since this is a critical point of the model.

Hypothesis

At a given length of the core, all parting sheets are assumed to have the same temperature. Therefore, each stream is supposed to exchange energy with the parting sheet only, regardless of the other streams. This hypothesis is known as the "Common Wall Temperature Hypothesis".

Main steps of the COLETH design procedure

The main steps of the COLETH procedure are now described:

– Composite curve construction – section determination. First the maximum duty section (and the corresponding Q^{MDS} duty) is defined.

– Distributor selection : The location of the distributors is defined by a procedure which takes into account the inlet and the outlet position related to the cooling curve, the allowable pressure drop and the stream flow rate.

– The following variables are fixed according to pressure considerations by the choice of the highest pressure among all streams: maximum core length (L^{max}), maximum number of layers, parting sheet thickness and bar width.

– Main Duty Section length calculation – fin selection. The maximum length of the Main Duty Section, $L^{MDS,max}$, is first computed using Eq. (1).

$$L^{MDS,max} = \frac{Q^{MDS}}{Q^{Tot}} \cdot L^{max} \qquad (1)$$

where L^{max} is the maximum value of the core length (evaluated according to pressure considerations), Q^{Tot} and Q^{MDS} are respectively the duties exchanged in the whole core and the Main Duty Section.

In each duty interval and for each stream, the Colburn factor (C_j) is evaluated. C_j is a function of the Reynolds number and the geometrical parameters of the fin: height, frequency, serration length and thickness. For the existing fins of the data bank, specific equations relating C_j to the Reynolds number and the geometrical parameters are used. These equations are either based on experimental data or on estimated data. For non standard fins, proprietary correlations are used. Non standard fins are generally involved during the optimization problem solution. For reasons of privacy, those correlations are not presented here. Classical experimental correlations are presented by Kays and London.[6]

Given the Colburn factor, the heat transfer coefficient is evaluated according to Eq. (2).

$$He = \Psi \cdot C_j \cdot \frac{m}{Ac} \cdot Cp \cdot (Pr)^{2/3} \qquad (2)$$

where Ψ is the area efficiency. In a layer, there are two types of area. The primary surface corresponds to the area between the stream and the separating plate. The secondary area refers to the fin area. The effectiveness of the secondary surface is reduced since the heat transferred must first be conducted along the fin. Thus, this efficiency is a function of the height and the thickness of the fin. In order to take into account non ideal arrangement, this efficiency is also affected by the ratio between the total number of cold streams and the total number of hot streams. For more details the interested reader should refer to Kays and London.[6]

Equation (2) is used for monophasic flows. For diphasic flows, specific correlations are available. One should note that the correlations used for diphasic flows involve discontinuities. This is of particular importance for the optimization problem solution.

The heat transfer resistance of stream i in interval j is given by Eq. (3):

$$R^{i,j} = \frac{1}{(He^{i,j} \cdot A_0^{i,j})} \qquad (3)$$

where $He^{i,j}$ is the heat transfer coefficient of stream i in interval j and $A_0^{i,j}$ is the exchange area per unit of length. The total heat transfer resistance is calculated for both the cold and the hot sides. Take for example the hot side, the transfer resistance is computed according to Eq. (4).

$$R^{hot,j} = \frac{1}{\sum_{\text{hot streams}} 1/R^{i,j}} \qquad (4)$$

Equation (5) gives the total heat transfer resistance.

$$R^{Tot,j} = R^{hot,j} + R^{cold,j} + R^{wall,j} \qquad (5)$$

where $R^{wall,j}$ is the resistance involved by the parting sheet and the different fins. $R^{wall,j}$ is a function of the thickness of the parting sheet and the fins.

The logarithmic difference of temperature, between the wall and the fluid, varies linearly with respect to the heat transfer resistance:

$$\Delta T_{lm}^{hot,j} = \Delta T_{lm}^{j} \cdot \frac{R^{hot,j}}{R^{tot,j}} \qquad (6)$$

In Eq. (6), ΔT^{j}_{lm} is the logarithmic mean temperature difference. Of course we have the same expression for cold streams.

Thus the length required by stream i in interval j is computed according to Eq. (7) as is shown for the hot stream in following example:

$$L^{i,j} = \frac{Q^{i,j}}{He^{i,j} A_0^{i,j} \Delta T_{lm}^{hot,j}} \qquad (7)$$

where $Q^{i,j}$ is the duty exchanged by stream i in interval j. For a given stream, the total length is:

$$L^{i} = \sum_{j=1}^{NI} L^{i,j} \qquad (8)$$

If the sizing mode is used, the length L^i of stream i is computed using the fin supplied by the user ($A_0^{i,j}$ is a known variable). If the automatic sizing mode is used, for stream i, the fins of the data bank are numbered. Heuristics allow partial listing only. If the calculated length (L^i) is lower than the maximum length ($L^{MDS,max}$), the current fin is accepted. With such a procedure, for stream i, several fins are selected. Finally, one fin is selected for each stream in such a way that the difference between the different lengths (L^i) is minimized. As said before this procedure is performed for the main duty section in order to select the fins. Afterwards the selected fins are generally used for the subsequent sections. In both cases, either sizing mode or automatic sizing mode, the duty requirement is satisfied.

In equation (7), for example, the heat capacity (Cp) is assumed to be constant. In most cases, if one considers the complete exchanger, duty is not a linear function of temperature. Then we have to introduce the duty intervals. As discussed in section 2, "Definitions", Cp are supposed to be constant in such intervals. Then, the selection of the number of intervals results in a compromise between the accuracy of the model and the computational load which is of particular importance for us. Thus, for this classical meshing problem, we have increased the number of intervals in order to achieve significantly constant results. For our applications, the number of intervals is approximately equal to 50.

Pressure drop calculation. For a given section, in each interval j and for each stream i, the friction factor (C_f) is evaluated. If the automatic sizing mode is used, the fins selected during the previous step (length calculation) are used. As for the Colburn factor, for existing fins of the data bank, specific equations relating C_f to the Reynolds number are used. Proprietary correlations are available when non standard fins are to be used. For a monophasic stream i in interval j, the pressure drop per unit of length is given by Eq. (9).

$$\Delta P^{i,j} = 4 \cdot C_f \cdot \frac{1}{D_h} \cdot \left(\frac{1}{2} \rho v^2\right) \qquad (9)$$

Empirical based correlations are available for diphasic flow. Then, for each stream, the total pressure drop is calculated according to Eq. (10):

$$\Delta P^{i} = \sum_{j=1}^{NI} \Delta P^{i,j} \qquad (10)$$

For stream i, the total pressure drop is evaluated adding the pressure drops of both the distribution and the exchange sections. An important point is that a fin is not rejected if the pressure drop constraint is not satisfied. Therefore the pressure drop requirements are not necessarily satisfied.

Conclusion

In conclusion, by using this procedure, the engineers are abble to solve the design problem. To face such a complex challenge, experience and know-how are of critical importance. Of course, pressure drop requirements must be satisfied. But many other constraints have to be considered. Moreover, in order to improve company profit, the manufacturing cost of the PFHE must be as low as possible.

Obviously, mathematical programming techniques are particularly adapted to the solution of this problem. So, our objective is to implement optimization algorithms in the COLETH code in order to improve the design, using a more systematic procedure.

Optimisation problem formulation

Introduction

The general formulation of the optimization problem is stated as follows :

$$\left.\begin{array}{l} \underset{x,y}{\text{Min}}\ f(x,y) \\ \text{s.t.} \\ g(x,y) = \leq 0 \\ x^{l} \leq x \leq x^{u} \\ y \in Y \end{array}\right\} \quad (P)$$

Where x and y are respectively the continuous and discrete variables, f is the objective function to be minimized and g is the set of constraints (equality or inequality).

We have adopted a two-level solution strategy. At the first level, the PFHE model is solved using the "sizing mode" of the COLETH procedure as described in the previous section. At the second level, the problem (P) is solved. Interest variables, which are computed at the model level and are implicit functions of the optimization variables, arise in the constraints and in the objective function. Because of the complexity of the COLETH software (COLETH has been developed during many years), it was impossible to select an equation oriented solution strategy.

Hypothesis

The computational load is a critical point for the NORDON company. Experience has proven that, if the computational load is too large, engineers will not use the optimization tool. They will go back to the classical design procedure.

Thus, in the present work, fin geometrical parameters and the number of layers are optimized in the main duty section only. In other sections, those parameters are fixed by the sizing COLETH procedure. This assumption is formulated according to computational time considerations. Since in most cases more than 60 % of the total duty is achieved in the main duty section, the solution of this problem gives a good approximation of the "real" optimum.

At the optimization level, the second main assumption is that we consider the number of layers as a continuous variable. The combinatorial aspect of the problem is decreased. Thus, the computational load is reduced.

Variables

Optimization variables (x and y) are described in figure 2. Continuous variables (x) are: core width (x^{CW}), distributor width (x^{DW}) and the number of layers (x^{L}). Discrete/Integer variables (y) are : core number (y^{CN}), fin height (y^{H}), fin thickness (y^{T}), fin frequency (y^{F}) and fin serration length (y^{S}). For straight perforated fins, y^{S} is the porosity of the fin. There are ($5.NS^{MDS}$ + ND + 2) optimization variables where NS^{MDS} is the number of streams in the main duty section and ND is the distributor number of the core. ($4.NS^{MDS}$ + 1) variables are discrete ones.

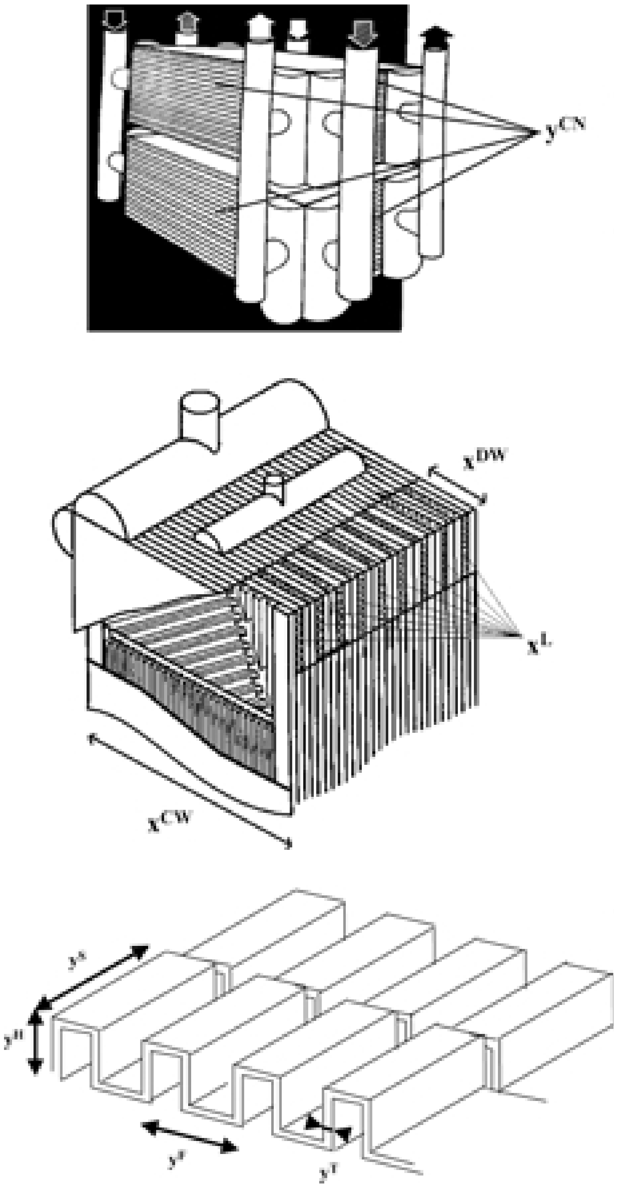

Fig. 2 – *Optimization variables*

Constraints

Optimization constraints are stated as follow:

Banking limit: In this formulation, the stream arrangement problem goes unsolved. Given an optimal design, the user has to define an optimal arrangement. Nevertheless, we introduce the following constraints : the ratio of hot to cold stream layers must be nearly equal to one (greater than 0.5 and lower than 2).

$$\frac{1}{2} \leq \frac{\sum_{i=1}^{NS^{hot,MDS}} x^{L,i}}{\sum_{i=1}^{NS^{cold,MDS}} x^{L,i}} \quad \text{and} \quad \frac{\sum_{i=1}^{NS^{hot,MDS}} x^{L,i}}{\sum_{i=1}^{NS^{cold,MDS}} x^{L,i}} \leq 2 \qquad (C_1 \text{ and } C_2)$$

$NS^{hot,MDS}$ and $NS^{cold,MDS}$ are the number of hot and cold layers in the Main Duty Section. These two constraints ensure that a feasible arrangement should be found in the final step of the design procedure.

Maximum stacking height: for mechanical and practical reasons the core stacking height (H) must be lower than a given value (H^{max}).

$$H(y^H, x^L) \le H^{max} \qquad (C_3)$$

with H^{max} = Min ($H1^{max}$, $H2^{max}(P)$)

where $H1^{max}$ is the absolute maximum according to the height of the brazing furnace and $H2^{max}$ (P) is the maximum value limited by mechanical considerations, which are design pressure dependant.

Maximum layer number: for mechanical reasons the total number of layers must be lower than a given value LN^{max}, which is pressure dependent.

$$\sum_{i=1}^{NS^{MDS}} x^{L,i} \le LN^{max}(P) \qquad (C_4)$$

Maximum width: for mechanical reasons the core width must be less than a given value CW^{max} which is pressure dependent.

$$x^{CW} \le CW^{max}(P) \qquad (C_5)$$

One should note that the optimization variable x^{CW} is bounded. The upper bound $x^{CW,\,u}$ corresponds to the absolute maximum value of the core width, which is restricted by the brazing furnace dimensions.

Maximum length: the core length CL must be less than a given value CL^{max} which is restricted by the brazing furnace dimensions.

$$CL(x, y) \le CL^{max} \qquad (C_6)$$

Fin manufacture feasibility: Since geometrical parameters of the fins can be optimized (and not chosen from the data bank among all existing fins), we must add constraints that ensure the manufacturing feasibility of the optimal fins. A proprietary correlation allows the calculation of the maximum fin frequency F^{max} as a function of the fin thickness and height.

$$y^{F,i} \le F^{max,i}(y^{H,i}, y^{T,i}) \quad i = 1, NS^{MDS} \qquad (C_7)$$

Maximum pressure: The optimal geometric parameters of the fins must be compatible with the stream pressures. Of course this is true for the existing fins of the database. But for the new optimal fins, the maximum pressure is computed as a function of its thickness, the frequency and the fin porosity.

$$P^i \le PF^{max,i}(y^{T,i}, y^{F,i}, y^{S,i}) \quad i = 1, NS^{MDS} \qquad (C_8)$$

Maximum erosion velocity: In order to avoid distributor erosion, the flow velocity in the distribution sections must be lower than a maximum value.

$$V^k(x^{CW}, x^{DW,k}, y^H, x^L, x^{CN}) \le V^{max,k} \qquad (C_9)$$

$$k = 1, ND$$

Minimal header size: The distributor width must be greater than a minimum value which is core width dependent.

$$DW^{min,k}(x^{CW}) \le x^{DW,k} \quad k = 1, ND \qquad (C_{10})$$

Maximal header size: The distributor width must be lower than a maximum value which is calculated according to the core width, the numbers of layers, the fin heights and the design pressure of the stream

$$x^{DW,k} \le DW^{max,k}(x^{CW}, x^L, y^H) \quad k = 1, ND \qquad (C_{11})$$

Header geometry: C10 and C11 are mechanical constraints. Header widths are also geometrically constrained: the sum of the different distributor widths must be lower than the core width (Figure 3). The ε tolerances ensure assembly feasibility.

– One "center" header (CEN) and one "end" header (END)

$$x^{DW}_{CEN} + x^{DW}_{END} - x^{CW} + \varepsilon_1 \le 0$$

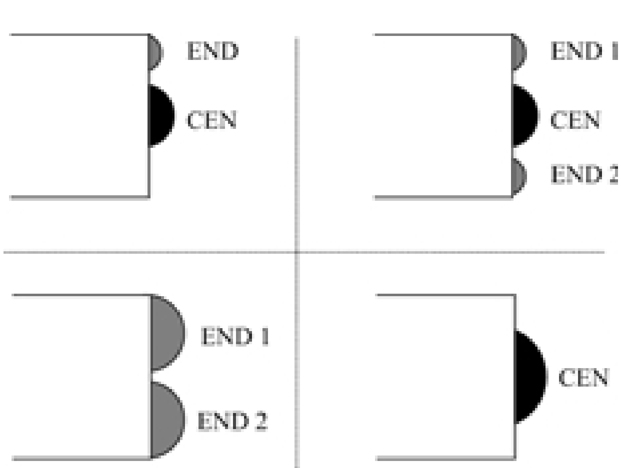

Fig. 3 – *Optimization constraints – header geometry*

– One "center" header (CEN) and two "end" headers (END1 and END2)

$$x_{CEN}^{DW} + x_{END1}^{DW} + x_{END2}^{DW} - x^{CW} + \varepsilon_2 \leq 0$$

– Two "end" headers (END1 and END2)

$$x_{END1}^{DW} + x_{END2}^{DW} - x^{CW} + \varepsilon_3 \leq 0$$

– One "center" header (CEN)

$$x_{CEN}^{DW} - x^{CW} + \varepsilon \leq 0 \qquad (C_{12})$$

Let MGEO be the number of geometric constraints. Geometrical constraints involved by the "side" distributor are not included.

Pressure drops: as said before the pressure drops for each flow must be lower than the maximum values.

$$\Delta P^{i} \leq \Delta P^{max,i} \quad i = 1, NS \qquad (C_{13})$$

One should note that the superscript i varies from 1 to NS which is the total number of streams (NS differs from NS^{MDS}, which is the number of streams in the maximum duty section).

The total number of constraints is: 6 + NS + $2NS^{MDS}$ + 3ND + MGEO. Constraints C_1, C_2, C_4, C_5, C_7 and C_{12} are explicit functions of the optimization variables. Constraints C_3, C_6, C_8, C_9, C_{10}, C_{11} and C_{13} involve variables of interest computed by the COLETH procedure at the model level.

It should be emphasized that the problem formulation is entirely automated. The user does not have to bother with variable or constraint definition, especially variables and constraints involved with the optimization of the distributors.

Objective function

Any variable computed by the COLETH procedure or any explicit function of the optimization variables can be used as the objective function. The more general one is the manufacturing cost by default. This manufacturing cost includes: raw material cost for the distribution and transfer fins, the headers, the nozzles, the edge bars and the parting and cap sheets; fin and header manufacture; core assembly; brazing time; testing procedures; and complete exchanger assembly. For reasons of privacy we are not able to give the detailed equations. The important point is that the manufacturing cost function is a non-linear, non-smooth function.

For more particular applications (airborne applications for example), other objective functions are available: total weight, total volume or total section area.

Optimisation problem solution

Relaxed NLP problem solution

The first way to solve this problem is to relax all optimization variables. Since variables are assumed to be continuous, the problem results in a Non Linear Programming Problem (NLP) which is solved using an SQP algorithm.[5]

A detailed description of the algorithm is outside the scope of this paper. However, for our application, the most interesting feature of this algorithm is the use of both line search and trust region strategies. This feature is supposed to promote global convergence behavior.

Even if both line search and trust region strategies are used, the SQP algorithm fails to find the global optimum (see illustrative examples in the next section). So, different initialization procedures have been tested in order to check whether the global optimum is reached:

– initial values are generated randomly in the interval which defines x: $[x^l;x^u]$,

– initial values are computed as the average value of the interval: $x^{ini} = \frac{x^l + x^u}{2}$,

– direct use of the results of the COLETH automatic sizing mode,

– modification of the results of the COLETH automatic sizing mode with respect to the general results observed in the test problems (to be seen in the following section): fin frequencies and heights are increased; fin thicknesses and serration lengths are decreased.

The direct use of the results of the COLETH automatic sizing mode yields the best results in most cases. Unexpectedly, the use of the modified results of the COLETH sizing mode does not achieve better convergence properties: of course, at the initial point, each optimization variable is generally closer to its optimal value. But the convergence path is not better: since we modify the geometrical parameters of the fins, constraints such as "fin manufacture feasibility" are now violated at the initial point. In the two illustrative examples, the third initialization procedure (direct use of the results of the COLETH automatic sizing mode) is used.

Such a tool is very interesting in order to outline the major trends of PFHE design and to perform sensitivity analysis (Lagrange multiplier values are available at the solution point). This can be an important element during the negotiation between the manufacturer and his client: if the maximum pressure drop is increased by one, the capital cost will be reduced by λ (Lagrange multiplier value). Check with your complete process! Another advantage is that the solution is achieved in a highly reduced computational time.

But at the solution point, the user has to consider the nearest acceptable value for each discrete variable in order to build a feasible PFHE. Moreover, there is no theoretical guarantee that the optimal solution of the MINLP problem is achieved. Such considerations induce us to consider MINLP algorithms.

MINLP Solution using branch and bound

A home made Branch and Bound algorithm has been used. This simple algorithm has been proven to be efficient for various process design problems.[7] For PFHE optimization, results are not so good. The first reason is that the combinatorial aspect of PFHE optimization is very important (see the second example). Consequently, the required computational time is inconsistent with an industrial use of the tool. The second reason is that, since we use the previously mentioned SQP algorithm, sub optimal solutions may arise while developing a branch. This is obvious when a new more restricted sub problem has a better solution than the node from which it branches.

MINLP Solution using Simulated Annealing

The main features of this optimization problem are as follows:

– The model of the PFHE is non continuous and non smooth

– The combinatorial aspect is great

– Objective function and constraints are non convex

– From a practical point of view, an oriented equation solution strategy is impossible

For these raisons, and for its simplicity, we have chosen the Simulated Annealing[8] algorithm. This is a non derivative stochastic method, with global convergence behavior. Since the considered optimization problem is a constrained one, a penalty function is minimized. The main drawback of this method is that the computational load is also great. Thus an important part of the present work is to reduce the computational load by optimizing the numerical parameters of the Simulated Annealing method. Two parameters have been especially optimized: its initial temperature (T0) and its temperature reduction coefficient (α).

Concerning the initial temperature, different methods have been tested. For a given example, optimization variables are first randomly moved from initial point. So an initial set of exchangers is built in the vicinity of the initial PFHE. The three most efficient methods are:

– T0 is a fraction of the minimum value of the objective function (f^{min}): T0 = $f^{min}/2$, $f^{min}/4$…

– T0 is evaluated according to the method proposed by Aarts and Korst[8]

– T0 is evaluated according to the method proposed by Maier[9]

In our case, the best results (global optimum with the lower computational charge) are generally achieved using the method proposed by Maier with an acceptation rate equal to 0.9.

If the temperature reduction coefficient is too low (the cooling rate is high), the algorithm obviously converges to a local minimum. If α is close to 1, the computational load is very high. It should be pointed out that, with α being close to 1, the algorithm is also trapped in a local minimum. In our case, the optimal value for the temperature reduction coefficient is 0.8.

While solving the MINLP problem by using Branch and Bound or Simulated Annealing, two strategies are possible:

– Given a set of standard discrete values for fin geometrical parameters, optimal values must be selected in order to minimize the objective function and satisfy the constraints. Therefore, the optimal fin is feasible (manufacturing constraints are necessarily satisfied), yet may not exist: NORDON will have to manufacture this new fin.

– Optimal fins must be selected among the existing fins. The Branch and Bound algorithm has been modified in the following way: while developing a branch, for each optimization variable, the set of possible discrete values is calculated accordingly to the previously constrained parameters and to the existing fin set. If the value set is empty, the sub problem is infeasible. So the Branch terminates. The Simulated annealing algorithm is modified in the same way.

Both strategies have been implemented.

Examples

Two industrial test problems are presented. Our main objective is to compare the results of the classical design procedure and the different optimal solutions.

First test problem

Consider the following illustrative example (Figure 4). There are one hot stream (H_1) and two cold streams (C_1 and C_2). One should note that the stream H_1 partially leaves the core at –116 °C. In

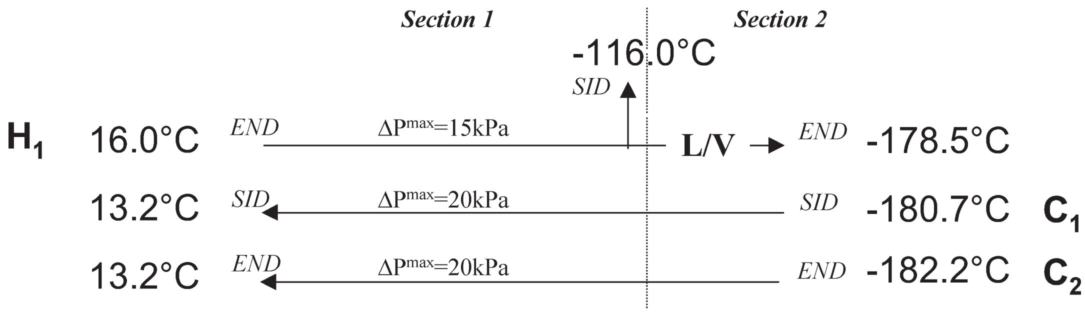

Fig. 4 – *First test problem*

the cold end of the exchanger, the hot stream is condensed resulting in a diphasic flow. This creates two sections. The Maximum Duty Section (section 1) includes three streams. The total number of variables is 24 and the total number of constraints is 38.

Manufacturing cost is minimized. Main results are presented in table 1. The commercial proposal is presented in the column entitled "Design". The solution of the relaxed NLP problem (using SQP algorithm) is presented in the column marked "SQP". Solutions of the original MINLP problem (using Branch and Bound or Simulated Annealing algorithm) are presented in the columns named "BB" and "SA".

Table 1 – *First test problem – results*

			Design	SQP	BB	SA
Optimization variables	x^{CW} [mm]		1 300	1 300	1 300	1 300
	y^{CN} [–]		1	1.1	1	1
	y^H [mm]	H_1	7.13	9.09	9.63	9.63
		C_1	9.63	9.63	9.63	9.63
		C_2	9.63	9.63	9.63	9.63
	y^T [mm]	H_1	0.20	0.20	0.20	0.20
		C_1	0.20	0.20	0.20	0.25
		C_2	0.20	0.20	0.20	0.25
	y^F [m^{-1}]	H_1	807.1	1000	771.7	897.6
		C_1	728.3	696.3	736.2	866.1
		C_2	728.3	691.2	736.2	866.1
	y^S	H_1	3.175	3.000	3.175	3.175
		C_1	15.875	3.329	9.525	9.525
		C_2	15.875	4.655	9.525	9.525
Interest variables	ΔP [kPa]	H_1	18	14	18	18
		C_1	14	14	14	14
		C_2	14	14	14	14
	Capital Cost*		100	92	93	85
	Total volume [m^3]		11.3	9.9	11.6	9.3
	CPU (Alpha Server 8200)		-	12'01"	11h21'	5h46'

*: basis is the capital cost of the commercial proposal

Let us compare the classical design and the relaxed optimum. Since the H_1 stream has the greatest allowable pressure drop, the fin frequency is increased (for better thermal effectiveness), the serration length is decreased (also for better thermal effectiveness), but fin height has to be augmented in order to satisfy the pressure drop constraint. For the other streams, the serration lengths are decreased (for better thermal effectiveness), but fin frequencies are also decreased in order to satisfy pressure drop requirements. The core width is always equal to the upper bound in order to satisfy pressure drop constraints. It should be emphasized that:

– The capital cost reduction is equal to 8 %,

– the CPU time is quite reasonable (12 minutes).

Let us compare the designed exchanger and the MINLP optimum using Simulated Annealing (column "SA"). The general tendencies are as follows: thicknesses and frequencies are increased, serration lengths are decreased. The result is improved thermal effectiveness. H_1 fin height is greatly increased in order to satisfy the pressure drop constraint. It should be emphasized that the capital cost is decreased by –15 %.

One should note that the optimal result using SA is better than the one using SQP. Actually, the relaxed optimum is theoretically better than the MINLP one. Solving the relaxed problem using SQP provides, on this example, a sub-optimal solution. Considering the pressure drop of stream H_1, one can note that the constraint is not saturated: the compromise between pressure drop and thermal effectiveness is not optimal (thermal effectiveness can be increased, and capital cost decreased, until H_1 pressure drop constraint is saturated). Of course the SA algorithm is much more time consuming than the SQP one.

The optimal solution using the Branch and Bound algorithm is also presented in the column called "BB". The algorithm is trapped in a poor local optimum: fin frequencies are too low. One should note the high computational load.

Considering the SA solution, the heat transferred in the main duty section is 68.3 % of the total duty. The length of the main duty section is 62.8 % of the total length. As discussed in section 3, "Hypothesis", the optimization of the main duty section gives us a good approximation of the "real" optimum (optimization of each section).

Second test problem

The second test problem is described in figure 5. This heat exchanger is a very challenging design problem: there is an intermediary outlet on the first hot stream (H_1). This stream is also redistributed in the third section. One should note that the layers occupied by stream H_2 in section 1 and 2 are occupied by the stream H_1 in section 3. This is an example of a duplex heat exchanger with redistribution. The maximum duty section is the second section. The total number of constraints is 49. There are 11 distributors and 28 optimization variables. 16 of them are discrete variables. Values of the discrete variables are shown in table 2. The combinatorial aspect of this example is quite important: 1 347 192 combinations. The main results are presented in table 3.

By solving the relaxed NLP optimization with an SQP algorithm, one achieves a 23 % manufacturing cost reduction. When using the SA algorithm, the manufacturing cost reduction is equal to

Table 2 – *Optimization variables – discrete values*

Variables	Units	Number of variables	Values	Number of values
Core Number (y^{CN})	-	1	1; 2; 4	3
Height (y^{H})	mm	3	3.53; 5.1; …	6
Thickness (y^{T})	mm	3	0.20; 0.25; …	7
Frequency (y^{F})	m^{-1}	3	393.7; 492.13; …	22
Serration Length (y^{S})	mm	3	3.175; 9.525; …	6

Table 3 – *Second test problem – results*

			Design	SQP	SA Capital Cost Min.	SA Volume Min
Optimization variables	x^{CW} [mm]		915	623	914	555
	y^{CN} [–]		1	1	1	1
	y^{H} [mm]	H_1	7.13	9.63	9.63	6.35
		H_2	7.13	9.11	3.53	3.53
		C_1	9.63	9.63	9.63	8.89
	y^{T} [mm]	H_1	0.40	0.21	0.25	0.25
		H_2	0.25	0.24	0.25	0.25
		C_1	0.20	0.20	0.20	0.25
	y^{F} [m–1]	H_1	600.4	970.6	866.1	866.1
		H_2	930	1000	984.3	984.3
		C_1	771.7	807.3	897.6	897.6
	y^{S}	H_1	3.175	3.333	3.175	3.175
		H_2	3.175	3.000	3.175	3.175
		C_1	3.175	3.734	3.175	9.525
Interest variables	ΔP [kPa]	H_1	15	15	15	15
		H_2	20	20	20	20
		C_1	20	20	20	20
	Capital Cost*		100	77	79	97
	Total volume [m3]		5.4	3.1	4.1	3.6

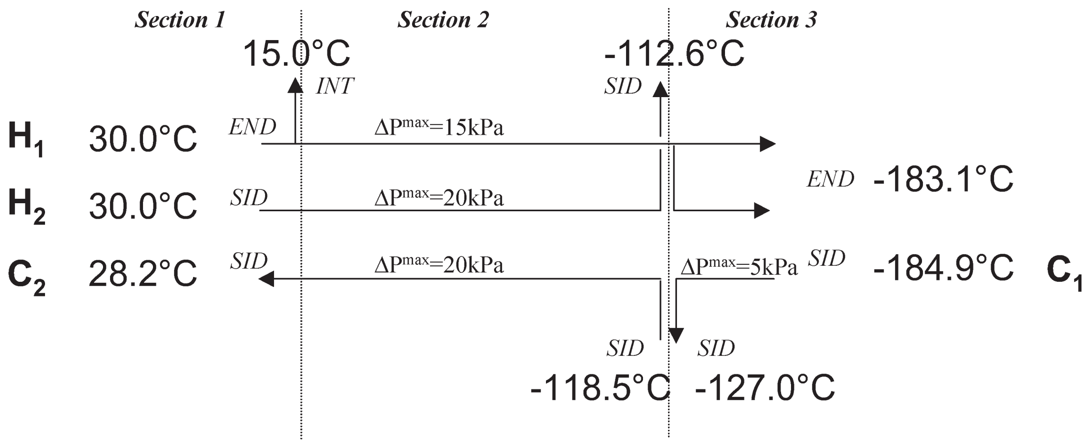

Fig. 5 – *Second Test Problem*

21 %. General trends are the following: by increasing fin frequency, thermal efficiency is improved. Pressure drop requirements are achieved by increasing the height (except for H_2, when using the SA!) and the thickness.

Results are also presented when minimizing the total volume of the exchanger. In this case, fin heights are decreased resulting in volume minimization. In addition, serration length of stream C_1 is increased in order to satisfy the pressure drop requirement.

Again, the main duty section is of particular importance in this example: the heat transferred in the main duty section is 66,7 % of the total duty. The length of the main duty section is 75 % of the total length.

Conclusion

The proposed work is an example of the industrial application of mathematical programming techniques. An efficient tool for computer aided design of plate fin heat exchangers is presented. Mathematical programming techniques are integrated in the COLETH program. The program allows for the optimization of fins (their height and thickness…), the core width and the widths of the various distributors. Also included are numerous design or operating constraints such as: pressure drops, maximum stacking height, maximum erosion velocity…. Various objective functions can be used too, like: manufacturing cost, total volume…. Most of the heat exchanger configurations can be optimized: intermediate by-products, redistribution…. The user can choose to relax the discrete variables if the SQP algorithm is used or he may consider a MINLP algorithm by using the Simulated Annealing or the Branch and Bound techniques. Two industrial examples are presented. Numerical considerations are discussed: the Simulated Annealing technique seems to achieve the best results with an acceptable computational load: manufacturing cost reduction varies from 15 to 21 %. General trends for PFHE design can also be outlined.

Future developments will include the optimization of the other heat exchanger sections. Indeed, in the present version, only the maximum duty section is optimized. The most challenging development is the optimization of the layer arrangement.

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Nomenclature

A – matrix of linearized constraints
A_0 – exchange area [m^2/m]
Ac – section area [m^2]
c – right end side of linearized constraints
Cf – friction factor
Cj – Colburn factor
CL – core length [m]
Cp – heat capacity [kJ/K/kg]
CW – core width [m]
Dh – hydraulic diameter [m]
DW – distributor width
f – objective function
F – fin frequency
g – optimization constraints
H – core height [m]
He – heat transfer coefficient [W/m^2/K]
L – length [m]
LN – total number of layers
m – mass flow rate [kg/s]
MGEO – number of geometric constraints.
ND – number of distributors
NS – number of streams
p – search direction
P – pressure
PF – fin design pressure
Pr – Prandlt number
Q – duty
R – heat transfer resistance [m.K/W]

T0 – initial temperature (SA)
v – stream velocity [m/s]
V – velocity in the distribution section [m/s]
x – continuous optimization variables
y – discrete optimization variables
Y – range space basis
Y – set of discrete values
α – temperature reduction coefficient
ΔP – pressure drop
ΔT – temperature difference
ε – tolerance
λ – Lagrange multiplier
ρ – mass density [kg/m^3]
Ψ – area efficiency

Superscript

CN – core number
cold – cold stream
CW – core width
DW – distributor width
F – fin frequency
H – fin height
hot – hot stream
i – stream
ini – initial
j – duty interval
k – distributor
l – lower bound
L – number of layers
max – maximum
MDS – main duty section
min – minimum
NI – number of duty intervals
S – fin Serration length
T – fin Thickness
Tot – total
u – upper bound
wall – parting sheet

Subscripts

CEN – "center" type distributor
END – "end" type distributor
lm – logarithmic mean