A Simple Hybrid Approach for the Synthesis of Heat Exchanger Networks Involving Internal Utility Exchangers

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This study aims to present a simple hybrid approach for synthesizing heat exchanger networks (HENs). The synthesis of HEN structures employs a genetic algorithm based on a modified node representation, closely resembling the stage-wise superstructure model. This modified representation allows for addressing the internal utility exchangers throughout the entire network. Despite the nonlinear nature of the model governing the continuous variables of each structure, a relatively simple linear formulation is developed to handle these variables. This novel formulation comprises a linear programming model for maximum energy recovery, a linear correction procedure (COP), and two search loops. The COP focuses on determining the optimal values of exchangers' heat loads and stream split ratios to reach the minimum total annual cost of the network. The study investigates four extensively studied medium- to large-scale networks from the literature. Despite the simplicity of the proposed approach, a comparison of results demonstrates its potential effectiveness.

Keywords
heat exchanger networks (HENs), internal utility exchangers, total annual cost (TAC), genetic algorithm (GA), linear programming (LP), correction procedure (COP)

Introduction

Heat integration through heat exchanger network (HEN) synthesis is recognized as a valuable tool for reducing energy consumption, related greenhouse gas emissions, and annual operating costs in processing plants. In recent decades, this topic has received increased attention due to the growing global demand for energy and a heightened focus on minimizing greenhouse gas production[1]. For these reasons, various approaches have been proposed to address these challenges[1–3]. The primary objective of HEN synthesis is to establish a network with an optimal arrangement of heat exchangers, resulting in the lowest total annual cost (TAC). The TAC encompasses heating and cooling utility costs, along with the annualized capital cost of the exchangers[4]. HEN problems are extremely challenging to formulate due to the simultaneous presence of integer and continuous variables, and the complexity of equations governing continuous variables. The challenges escalate with an increasing number of process streams increases, introducing heightened sophistication. Integer variables in the model pertain to the arrangement of exchangers in the network, while the continuous variables are associated with stream temperatures, exchanger heat loads and their areas, as well as stream-splitting ratios. Consequently, achieving the overall optimal solution to these problems, especially in large-scale networks, becomes more complex[4,5].

Based on the literature, HEN synthesis methods can essentially be classified as sequential or simultaneous, requiring deterministic or non-deterministic approaches to achieve optimal solutions[3,6,7]. In sequential methods, which fall into two subcategories: thermodynamic methods and sequential mathematical programming methods, the HEN is decomposed into a series of sub-problems in order to reduce the corresponding computational requirements. Pinch technology, a well-known method for HEN synthesis, considers thermodynamic laws and has a broad range of applications[8,9]. In this method, to reach maximum energy recovery (MER), the network is divided into two sub-networks above and below the pinch point, and each sub-network is optimized individually. Despite its flexibility and ability to provide a good view of the problem, this method relies on the designer's experience and does not guarantee optimal solutions in the simultaneous overall optimization of energy consumption and investment costs, especially for large-scale networks, as it is focused on MER[10].

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The sequential mathematical programming methods consist of three steps: firstly, obtaining the lowest utility costs through the application of a linear programming (LP) formulation, based either on a transportation model[11] or on an LP version of a transshipment model[12]. Secondly, based on a transshipment model, the minimum number of exchangers is handled through the mixed integer linear programming (MILP) formulation[12]. Lastly, the computation of the minimum investment cost is performed by solving the non-linear programming (NLP) model[13]. Although further development of this approach has yielded potential optimal solutions[14,15], its performance is still considered inferior to that of simultaneous methods due to the complexity of the combined HEN synthesis[5].

Simultaneous methods, eliminating the need for problem division, are developed to address the shortcomings of sequential methods and to reach better solutions. These methods include mixed integer non-linear programming (MINLP) models. Two types of models have been developed to solve HEN problems thus far: hyperstructure models[16,17] and stage-wise superstructure (SWS) models[18,19].

The SWS model permits connections between hot and cold streams at each stage, making it more popular than the hyperstructure model. Improvements have been made to the SWS model over the last few years to enhance its performance. Modifications such as non-isothermal mixing and consideration of utility exchangers inside each stage[20,21], and the possibility of creating sub-splits and cross-flow in each stage of the SWS[20], are among these enhancements.

Since HEN problems, both in sequential and simultaneous formulations, belong to the class of non-deterministic polynomial-time hard problems (Np-hard)[22], they are unlikely to be solvable in the form of an accurate and computationally efficient solution. This fact also emphasizes the inefficiency of deterministic methods, including generalized Benders decomposition (GBD), branch and bound (BB), and outer approximation (OA)[5] methods.

Thus, stochastic and meta-heuristic methods are favored over deterministic ones, despite their relatively long computing times, because they are not limited by the non-linearity, non-convexity, and discontinuity of HEN synthesis models. Simulated annealing (SA)[23], tabu search (TS)[24], harmony search (HS)[25], particle swarm optimization (PSO)[26], genetic algorithms (GA)[4,5,27–31], differential evolution (DE) algorithm[32], cuckoo search algorithm (CSA)[33], ant colony optimization (ACO)[34], random walk algorithm with compulsive evolution (RWCE)[35] and imperialist competitive algorithm (ICA)[36] are among these methods. A comprehensive review of them is given in the work of Toimil and Gómez[2].

However, the simultaneous treatment of continuous and integer variables of HEN problems by stochastic methods does not guarantee the achievement of an optimal solution[2,7]. Consequently, in recent years, the tendency to employ two-level approaches to deal with HEN synthesis problems has increased since they use different methods to handle continuous and integer variables at different levels[5,7].

Levin[27] proposed a two-level GA-NLP hybrid approach in which GA was utilized to generate HEN structures based on node representation while an NLP model was employed to handle discrete variables based on maximum energy recovery (MER). Although a constant minimum approach temperature was considered in the network, the formulation of the model to solve HEN problems was not complex. Khorasany and Fesanghary[25] presented a hybrid approach to tackle HEN synthesis that incorporated HS and sequential quadratic programming (SQP). Toffolo[37] applied a combination of SQP and evolutionary algorithms (EA) to find HEN solutions in an unconstrained topology. To address HEN synthesis, Zhaoyi *et al.*[38] presented a modified two-level simultaneous step synthesis approach, based on GA and PSO. In this approach, HEN structures were produced by GA at the high level, while the continuous variables were optimized at the low level by PSO. Peng and Cui[39] suggested a two-level SA approach for the solution of HEN problems. Using the ACO_R algorithm, Porfarhadi Miankoh and Shafiei[34] determined the feasible range of HEN synthesis. This approach had the disadvantage of not using stream splitting. Pavão *et al.*[20,40] presented a comprehensive approach for handling HEN problems, based on a hybrid SA-RFO method, and considering a different stage-wise superstructure model. The advantages of this approach included the use of serial process and utility exchangers on stream splits, and the possibility of creating sub-splits and cross-flows in each part of the HEN superstructure, making it more efficient at generating novel solutions. Bao *et al.*[21] suggested a different approach to HEN synthesis by incorporating an optimum-protection strategy into the RWCE. Their network configuration was based on the modified stage-wise superstructure presented by Pavão *et al.*[41], which permitted utility exchangers to be considered inside the network. Furthermore, a leader-follower optimization algorithm was incorporated into this approach to reduce calculation time, especially for networks with stream splitting.

Rathjens and Fieg[5] utilized a revolutionary hybrid approach called structure identification and change of reference system (SIR) – hybrid genetic algorithm (HGA) for network synthesis. The HGA, derived from earlier work[10,42], was a combination of

GA with various optimization algorithms such as a local optimization algorithm, and a structure control algorithm. This combination was used for the stochastic global search of the stage-wise superstructure. The SIR was also developed for the local optimization of HEN structures, providing the possibility of a better search for continuous variables in the network by reducing the number of optimization variables. Zamora *et al.*[43] addressed a two-step method to optimize and relocate utility exchangers to improve base HEN design. The first step aimed to search for the most appropriate values of design operating variables with the help of an NLP model previously developed by Núñez-Serna and Zamora[44]. In the second step, better solutions for the TAC of the network were found by rearranging the hot and cold utility exchangers in the base network design. Five heuristic utility relocation strategies were proposed in this study to identify suitable potential locations for utility exchangers. Xu *et al.*[45] employed a radically different approach called optimization route arrangement, consisting of multiple modules, to solve the problems of HEN synthesis. These modules were primarily based on RWCE with a variety of functions. They were used to achieve a balance between global and local optimal search as well as to solve optimization problems involving integers and continuous variables. Another significant feature of this approach was the use of the nodes-based nonstructural model (NNM) to produce HENs. The NNM was capable of producing more diverse networks than the stage-wise superstructure model. Despite the extreme efficiency of this approach, the calculation time was very high, as it required more than 79 hours to manage medium to large networks.

Examining various approaches proposed for the synthesis of HENs indicates that the complexity of reaching optimal solutions is still a challenge for most of them. Therefore, this study seeks to address a relatively simple, linear approach to HEN problems that guarantees better optimal solutions. It comprises a combination of GA, LP formulation, a correction procedure (COP), and two search loops. The GA generates different networks during the optimization process and identifies the best ones. The structures are produced using a modified node representation, allowing for the bifurcation of process streams upon entering the network and the implementation of internal utility exchangers. The model governing the continuous variables of any production network by the GA is generally a complex NLP model. To reduce the complexity of this model, a relatively simple linear formulation is developed in this research to handle these variables. This new method includes an LP model, a linear correction procedure, and two search loops.

The LP is formulated based on MER and determines stream temperatures and heat loads of exchangers. To achieve the lowest TAC, the heat loads and split ratios of the process exchangers are modified in a linear correction procedure. Due to the relatively simple formulation compared to the NLP model governing continuous variables, the likelihood of finding optimal solutions increases.

Problem definition

The synthesis of HEN problems is described as follows: A set of *NH* hot process streams needs to be cooled down and a set of *NC* cold process streams needs to be heated up. The supply temperatures, T^{in} (°C), target temperature, T^{out} (°C), heat transfer coefficients, h (kW m^{-2} K^{-1}), and heat capacity flow rates, W (kW K^{-1}), of these streams are specified. Additionally, a set of hot and cold utility (HU and CU) streams with defined temperature levels and heat transfer coefficients is provided. In order to calculate the network's TAC as the objective function, economic parameters, including the heat exchangers' purchase cost and the cost of supplying hot and cold utilities (C_{HU} and C_{CU}), should be provided.

Methodology

Fig. 1 illustrates the overall procedure of the proposed approach. As shown in the figure, optimization begins with an initial population produced randomly by the GA. In the next step, networks are evaluated by an LP, formulated based on MER. The LP section consists of a linear objective function for MER, and a search loop to find the appropriate split ratios (y) within the network. The split ratios are bound [0.01–0.99].

In addition, a reasonable value of 10 °C for ΔT_{min} is considered at this stage for each network. Although ΔT_{min} plays a significant role in the network's TAC, finding its appropriate value is not essential at this point, as its optimal value will be determined at the end of the optimization process. Following the LP results, the exchangers' areas will be available, allowing the TAC to be calculated. This TAC is used as a criterion for optimizing the split ratios in the search loop in this part of the proposed approach.

In the following, the heat loads of internal exchangers, split ratios, and streams' temperatures are modified through a linear correction procedure

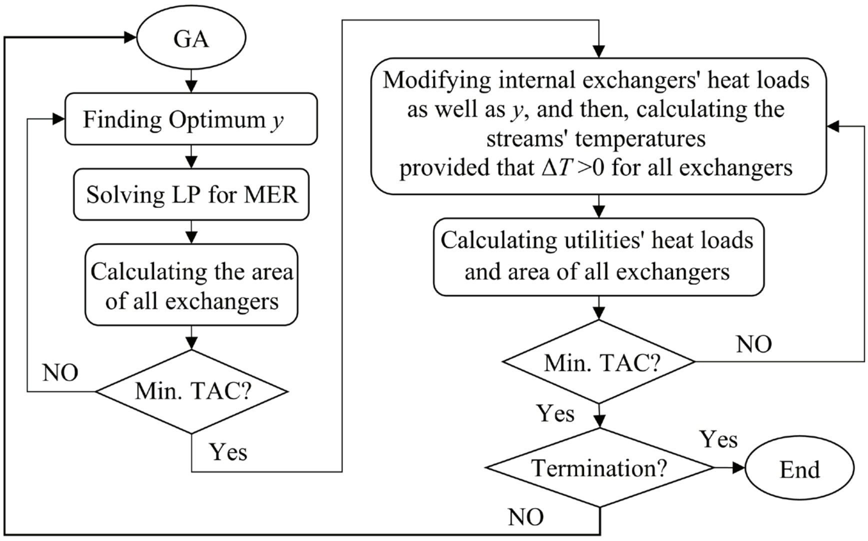


Fig. 1 – *Overall procedure of HENs synthesis*

(COP) to achieve better TAC. The condition for accepting these modifications is the positive approach temperature at the hot or cold end of exchangers, ΔT. Then the heat loads of utilities and the areas of all exchangers are calculated. Eventually, the TAC is computed as network fitness. In the COP section, the calculated final TAC is considered a criterion for correcting the thermal load of internal exchangers and split ratios in the external search loop. In addition, the minimum value of ΔT of exchangers is determined as ΔT_{min} of the network.

After the determination of the TAC of the first generation, the next generations are produced using the GA operators, reproduction, crossover, and mutation. These operators are described in detail elsewhere[4]. The overall algorithm is repeated for a specific number of generations. The following subsections describe the elements of the proposed approach outlined in Fig. 1.

Structural optimization

The GA is responsible for generating and identifying optimal HEN structures in this research. Each structure in this algorithm is considered a chromosome formed by connecting multiple genes. The number of genes (NG) in each network depends on the size of the problem. The address of heat exchangers in each gene is expressed as an array of several integer numbers. Producing network structures in this way is similar to the SWS proposed by Yee and Grossmann[19], but only a part of the feasible connections of the SWS is applied.

Fig. 2 illustrates all kinds of internal matches considered for each gene, including process exchangers and internal utility exchangers. Each gene contains a minimum of one exchanger and a maximum of three exchangers. In this work, the mixing of streams is non-isothermal. It should be noted that, increasing the number of branches will, on the one hand, increase the variety of HEN structures generated by the GA; on the other hand, it will complicate the structures and increase the time required to find optimal solutions. Thus, in this study, a maximum of three splits is given for the streams within a gene. Further, two additional items are implemented to increase the variety of production structures, as follows:

– Bifurcation of some process streams upon entry into the network and reconnection upon exit.

This will be useful only for streams with a high heat capacity flow rate and a large difference in supply and target temperatures. The bifurcation and mixing of these streams take place before and after the first and last genes of each structure. These genes are referred to as zero and NG+1 genes, respectively. Therefore, they contain no exchangers.

– Utilization of hot and cold utility exchangers within the network and traditionally at the ends of streams.

For this purpose, the hot and cold utility streams are considered fictitious process streams with stream numbers of NH+1 and NC+1, respectively. The numbering can be extended to multiple utility streams. The heat capacity flow rate of these fictitious process streams is considered negligible. It should be noted that the inlet and outlet temperatures of these streams to and from each exchanger are known according to the data of each case study.

In this way, the HEN structures produced by the GA will become much more diverse than in the work of Feyli *et al.*[4] Therefore, the probability of achieving the final optimal structure will increase.

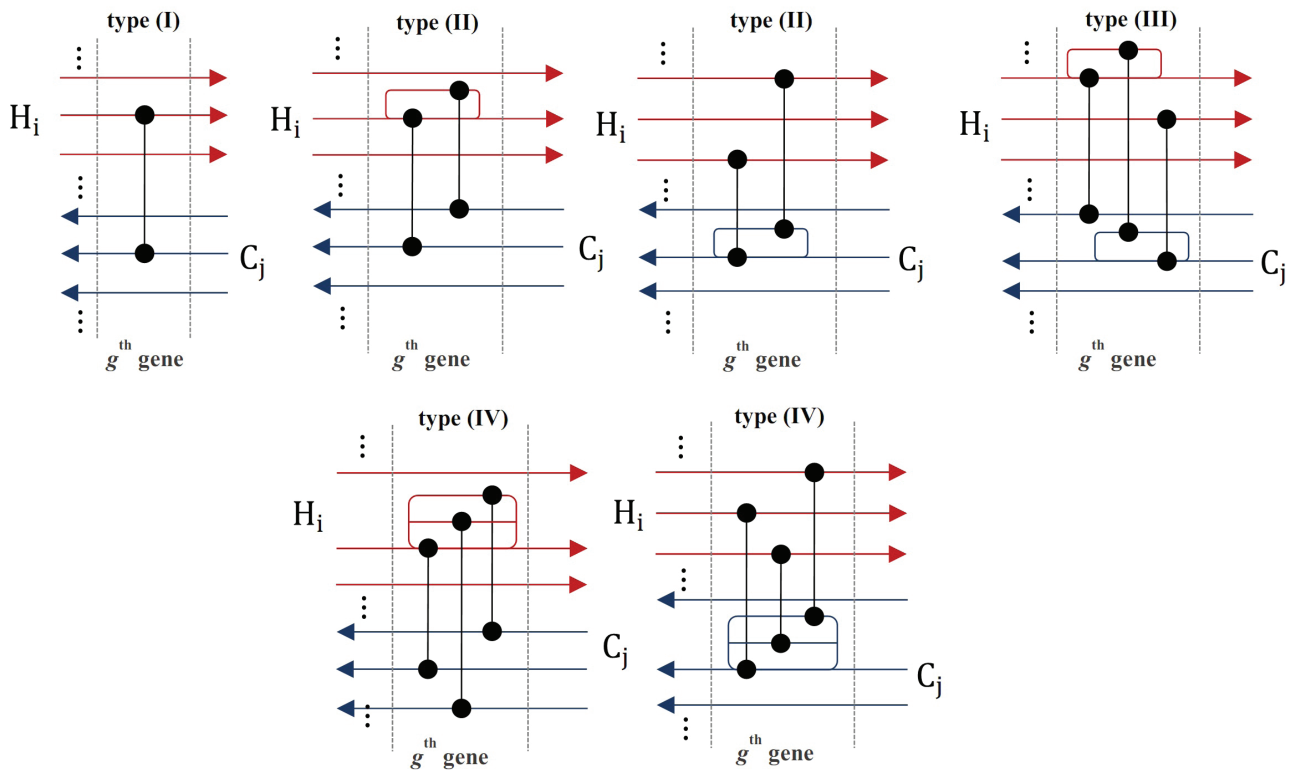


Fig. 2 – *Types of possible connections in each gene*

To express the location of all internal exchangers in each network, an exchanger address matrix (EAM), extended in comparison to the one presented by Feyli *et al.*[4], is defined, in which each row represents an exchanger's address. An example of a typical network is illustrated in Fig. 3a, with four genes, two hot streams (*NH*=2), three cold streams (*NC*=3), seven process exchangers, one internal hot, and one internal cold utility exchanger. Also, its EAM is shown in Fig. 3b.

In the EAM, the first column represents the hot stream number [1–*NH*], the second one is the hot stream main branch number upon entering the network (which is 1 or 2), and the third is the node number of the hot stream (which is 1, 2 or 3). The 4^{th}, 5^{th}, and 6^{th} columns contain similar numbers for cold streams, while the last column represents the gene number. If the stream entering the network does not branch, the number corresponding to the stream main branch number (i.e., the numbers in the 2^{nd} and 5^{th} column) is considered zero.

It should be noted that utility exchangers, which are employed at the end of streams when necessary, are not included in the EAM.

As observed in Fig. 3b, the hot stream number corresponding to E_2 exchanger and the cold stream number corresponding to E_6 exchanger, in the EAM (i.e., the hot and cold internal utility exchangers) are indicated by 3 and 4, respectively. Fig. 3 also illustrates the possibility of making use of utility exchangers in different stream branches and/or on the mainstream in a network. This representation of HEN structures is especially suited to GA operators and always leads to the production of feasible networks.

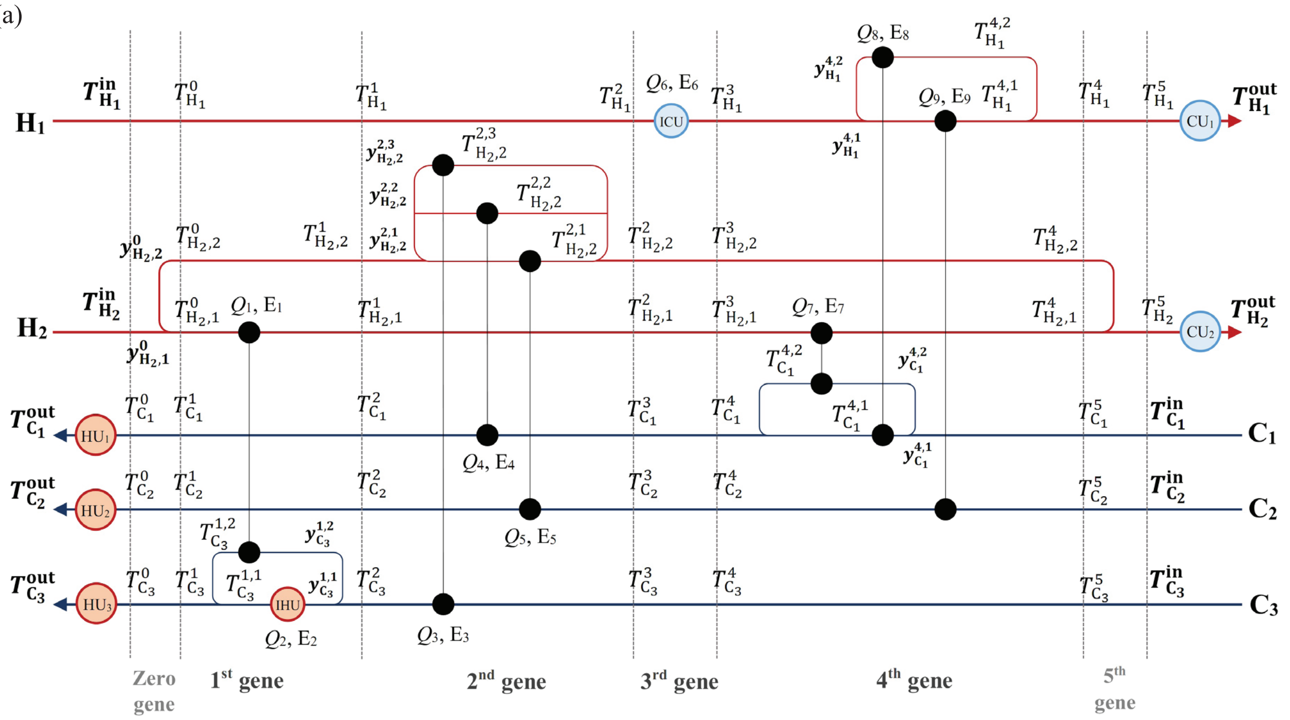
LP procedure

The LP model was formulated utilizing a multi-objective function to relax some exchangers from pinching at ΔT_{min} while achieving MER as follows:

$$\text{Max.} \sum_{k=1}^{NPE} Q_k + \left[\sum_{k=1}^{NIE} \left(\Delta T_{k,1} + \Delta T_{k,2} \right) \right] / SF \qquad (1)$$

Where Q_k is the process exchangers' heat loads, *NPE* and *NIE*, represent the number of process exchangers and internal exchangers (i.e., process and internal utility) in a network, and ΔT is the approach temperature at the hot or cold end of the exchangers. Also, *SF* is a scaling factor that must be large enough to ensure that the second term in Equation (1) does not affect the objective, which is MER.

(a)



(b)

$$
\mathbf{EAM} = \begin{bmatrix}
E_1: 2\ 1\ 1\ 3\ 0\ 2\ 1 \\
\boldsymbol{E_2}: \mathbf{3}\ 0\ 1\ 3\ 0\ 1\ 1 \\
E_3: 2\ 2\ 3\ 3\ 0\ 1\ 2 \\
E_4: 2\ 2\ 2\ 1\ 0\ 1\ 2 \\
E_5: 2\ 2\ 1\ 2\ 0\ 1\ 2 \\
\boldsymbol{E_6}: 1\ 0\ 1\ \mathbf{4}\ 0\ 1\ 3 \\
E_7: 2\ 1\ 1\ 1\ 0\ 2\ 4 \\
E_8: 1\ 0\ 2\ 1\ 0\ 1\ 4 \\
E_9: 1\ 0\ 1\ 2\ 0\ 1\ 4
\end{bmatrix}
$$

Fig. 3 – *(a) HEN with nine exchangers, (b) its EAM*

The constraints of the LP model according to the network shown in Figure (3a) are as follows:

– Energy balance for internal process exchangers (nonlinear if splitting occurs):

$$
\begin{aligned}
&E_1: T^0_{H_2,1} - Q_1 / \left(y^0_{H_2,1} W_{H_2}\right) - T^1_{H_2,1} = 0,\ \ T^2_{C_3} + Q_1 / \left(y^{1,2}_{C_3} W_{C_3}\right) - T^{1,2}_{C_3} = 0 \\
&E_3: T^1_{H_2,2} - Q_3 / \left(y^0_{H_2,2}\, y^{2,3}_{H_2,2} W_{H_2}\right) - T^{2,3}_{H_2,2} = 0,\ \ T^3_{C_3} + Q_3 / W_{C_3} - T^2_{C_3} = 0 \\
&E_8: T^3_{H_1} - Q_8 / \left(y^{4,2}_{H_1} W_{H_1}\right) - T^{4,2}_{H_1} = 0,\ \ T^5_{C_1} + Q_8 / \left(y^{4,1}_{C_1} W_{C_1}\right) - T^{4,1}_{C_1} = 0
\end{aligned}
\tag{2}
$$

For other process exchangers the same relations must be written.

– Energy balance for internal utility exchangers (nonlinear equations if splitting occurs):

$$
\begin{aligned}
&E_2: T^{in}_{HU} \text{ and } T^{out}_{HU} = \text{known},\ T^2_{C_3} + Q_2 / \left(y^{1,1}_{C_3} W_{C_3}\right) - T^{1,1}_{C_3} = 0 \\
&E_6: T^2_{H_1} - Q_6 / W_{H_1} - T^3_{H_1} = 0,\ T^{in}_{CU} \text{ and } T^{out}_{CU} = \text{known}
\end{aligned}
\tag{3}
$$

– Energy balance for utility exchangers at stream ends (linear equations):

$$
\begin{aligned}
&CU_1: T^5_{H_1} - Q_{CU_1} / W_{H_1} - T^{out}_{H_1} = 0 \\
&HU_1: T^0_{C_1} + Q_{HU_1} / W_{C_1} - T^{out}_{C_1} = 0
\end{aligned}
\tag{4}
$$

For other utility exchangers at stream ends, the same relations must be written.

– Energy balance at mixing points (nonlinear equations):

$$\begin{gathered} T_{H_1}^{4} = y_{H_1}^{4,1} T_{H_1}^{4,1} + y_{H_1}^{4,2} T_{H_1}^{4,2} \\ T_{H_2,2}^{2} = y_{H_2,2}^{2,1} T_{H_2,2}^{2,1} + y_{H_2,2}^{2,2} T_{H_2,2}^{2,2} + y_{H_2,2}^{2,3} T_{H_2,2}^{2,3} \\ T_{H_2}^{5} = y_{H_2,1}^{0} T_{H_2,1}^{4} + y_{H_2,2}^{0} T_{H_2,2}^{4} \\ T_{C_1}^{4} = y_{C_1}^{4,1} T_{C_1}^{4,1} + y_{C_1}^{4,2} T_{C_1}^{4,2} \\ T_{C_3}^{1} = y_{C_3}^{1,1} T_{C_3}^{1,1} + y_{C_3}^{1,2} T_{C_3}^{1,2} \end{gathered} \tag{5}$$

– Mass balance on splitters (linear equations):

$$\begin{gathered} y_{H_2,1}^{0} + y_{H_2,2}^{0} = 1 \\ y_{H_2,2}^{2,1} + y_{H_2,2}^{2,2} + y_{H_2,2}^{2,3} = 1 \\ y_{C_3}^{1,1} + y_{C_3}^{1,2} = 1 \end{gathered} \tag{6}$$

Similar relationships must be written for other splitters.

– Monotonic decrease or increase in temperature in streams (linear inequalities):

$$\begin{gathered} H_1 : T_{H_1}^{in} \geq T_{H_1}^{0} \geq T_{H_1}^{1} \geq \ldots \geq T_{H_1}^{5} \geq T_{H_1}^{out} \\ H_2 : T_{H_2}^{in} \begin{cases} \geq T_{H_2,1}^{0} \geq T_{H_2,1}^{1} \geq \ldots \geq T_{H_2,1}^{4} \\ \geq T_{H_2,2}^{0} \geq T_{H_2,2}^{1} \geq \ldots \geq T_{H_2,2}^{4} \end{cases} , mix\left(T_{H_2,1}^{4}, T_{H_2,2}^{4}\right) \geq T_{H_2}^{out} \\ C_1 : T_{C_1}^{in} \leq T_{C_1}^{5} \leq T_{C_1}^{4} \leq \ldots \leq T_{C_1}^{0} \leq T_{C_1}^{out} \end{gathered} \tag{7}$$

For other streams, the same relations must be written.

– Minimum approach temperature constraints (linear inequalities):

$$\begin{gathered} E_1 : \Delta T_{1,1} = T_{H_2,1}^{0} - T_{C_3}^{1,2} \geq \Delta T_{min},\ \Delta T_{1,2} = T_{H_2,1}^{1} - T_{C_3}^{2} \geq \Delta T_{min} \\ E_2 : \Delta T_{2,1} = T_{HU}^{in} - T_{C_3}^{1,1} \geq \Delta T_{min},\ \Delta T_{2,2} = T_{HU}^{out} - T_{C_3}^{2} \geq \Delta T_{min} \\ E_6 : \Delta T_{6,1} = T_{H_1}^{2} - T_{CU}^{out} \geq \Delta T_{min},\ \Delta T_{6,2} = T_{H_1}^{3} - T_{CU}^{in} \geq \Delta T_{min} \\ CU_1 : \Delta T_{CU_1,1} = T_{H_1}^{5} - T_{CU}^{out} \geq \Delta T_{min},\ \Delta T_{CU_1,2} = T_{H_1}^{out} - T_{CU}^{in} \geq \Delta T_{min} \\ HU_1 : \Delta T_{HU_1,1} = T_{HU}^{in} - T_{C_1}^{out} \geq \Delta T_{min},\ \Delta T_{HU_1,2} = T_{HU}^{out} - T_{C_1}^{0} \geq \Delta T_{min} \end{gathered} \tag{8}$$

Similar relationships must be written for other internal exchangers and utility ones at the stream ends.

In the above equations, $T_{H_i}^{in}$, $T_{C_j}^{in}$, $T_{H_i}^{out}$ and $T_{C_j}^{out}$ represent the supply and target temperatures of the i^{th} hot and j^{th} cold streams, respectively. T_{HU}^{in}, T_{CU}^{in}, T_{HU}^{out} and T_{CU}^{out} represent the supply and target temperatures of the hot and cold utilities, respectively. $T_{H_i}^{g}$ and $T_{C_j}^{g}$ respectively indicate the output of i^{th} hot and j^{th} cold stream from the g^{th} gene. $T_{H_i}^{g,s}$ and $T_{C_j}^{g,s}$ respectively represent the i^{th} hot and j^{th} cold stream of the s^{th} split located in the g^{th} gene. $T_{H_i,b}^{g}$ and $T_{C_j,b}^{g}$ indicate the output temperature of the b^{th} branch of the hot and cold streams from the g^{th} gene, respectively. $T_{H_i,b}^{g,s}$ and $T_{C_j,b}^{g,s}$ respectively represent the temperatures of the hot and cold streams of the s^{th} split of the b^{th} branch located in the g^{th} gene. $y_{H_i,b}^{g}$ and $y_{C_j,b}^{g}$ denote the ratio of the b^{th} branch of the i^{th} hot stream and j^{th} cold stream, respectively, upon entering the network (i.e., g=0 for hot stream and g= NG+1 for cold one). $y_{H_i}^{g,s}$ and $y_{C_j}^{g,s}$ respectively denote the ratio of the s^{th} split of the i^{th} hot stream and j^{th} cold stream located in the g^{th} gene. $y_{H_i,b}^{g,s}$ and $y_{C_j,b}^{g,s}$ represent the ratio of the s^{th} split from the b^{th} branch of the i^{th} hot stream and j^{th} cold stream, respectively, located in the g^{th} gene. Q_{CU_i} and Q_{HU_j} denote the heat load of cold utility and hot utility exchangers, respectively, and W is the heat capacity flow rate. Furthermore, NG, NH, NC, and NIE

represent the number of genes in each HEN, the number of hot process streams, the number of process cold streams, and the number of internal exchangers (including process and internal utility exchangers), respectively. (Note: $g \in NG$, $i \in NH$, $j \in NC$, $k \in NIE$, $s \in \{1,2,3\}$ and $b \in \{1,2\}$)

The main objective function

The main objective function of HEN synthesis to minimize TAC is expressed as follows:

$$\text{Min. TAC} = AF + \left\{ \sum_{k=1}^{NPE} (C_{\text{fixp}} + C_{\text{Ap}} A_k^{\beta}) + \sum_{i=1}^{NH} (C_{\text{fixc}} + C_{\text{Ac}} A_{\text{CU}_i}^{\beta}) + \sum_{j=1}^{NC} (C_{\text{fixh}} + C_{\text{Ah}} A_{\text{HU}_j}^{\beta}) \right\} + \sum_{i=1}^{NH} C_{\text{CU}} Q_{\text{CU}_i} + \sum_{j=1}^{NC} C_{\text{HU}} Q_{\text{HU}_j}$$

$$AF = \begin{cases} \dfrac{(1+I)^n - 1}{I(1+I)^n} if\ I \neq 0 \\ 1 if\ I = 0 \end{cases} \tag{9}$$

where, C_{fixp}, C_{fixc}, and C_{fixh} denote the fixed capital cost, and C_{Ap}, C_{Ac}, and C_{Ah} are the area coefficients for processes, cold and hot utility exchangers, respectively. β is the area cost exponent. C_{CU} and C_{HU} are the cold and hot utility cost coefficients, respectively. AF, n, and I indicate the annualization factor, plant lifetime, and interest rate, respectively. A_{k}, A_{CU_i}, and A_{HU_j} are areas of the process, cold, and hot utility exchangers, which are defined according to Equation (10):

$$A_k = Q_k / \left(U_k\, LMTD_{\text{P}_k}\right),\ U_k = \left(\frac{h_{\text{H}_i}\, h_{\text{C}_j}}{h_{\text{H}_i} + h_{\text{C}_j}}\right) \tag{10}$$

$$A_{\text{CU}_i} = Q_{\text{CU}_i} / \left(U_{\text{CU}_i}\, LMTD_{\text{CU}_i}\right),\ U_{\text{CU}_i} = \left(\frac{h_{\text{H}_i}\, h_{\text{CU}_i}}{h_{\text{H}_i} + h_{\text{CU}_i}}\right)$$

$$A_{\text{HU}_j} = Q_{\text{HU}_j} / \left(U_{\text{HU}_j}\, LMTD_{\text{HU}_j}\right),\ U_{\text{HU}_j} = \left(\frac{h_{\text{HU}_j}\, h_{\text{C}_j}}{h_{\text{HU}_j} + h_{\text{C}_j}}\right)$$

where, h and U refer to the streams' film heat transfer coefficients and the exchangers' overall heat transfer coefficient, respectively, and $LMTD$ is the logarithmic mean temperature difference, which is defined according to Equation (11):

$$\begin{cases} LMTD = \left(\Delta T_1 - \Delta T_2\right) / \ln\left(\Delta T_1 / \Delta T_2\right), & if\ \Delta T_1 \neq \Delta T_2 \\ LMTD = \left(\Delta T_1 + \Delta T_2\right) / 2, & if\ \Delta T_1 = \Delta T_2 \end{cases} \tag{11}$$

Although, the relationships reported for utility and process exchangers are commonly considered the same in most literature, in this research, according to Equation 9, these relationships are considered different for each type of exchanger.

One is able to calculate the exchangers' areas and subsequently the TAC after utilization of the LP. At this stage of the proposed model, attaining the minimum TAC is used as a criterion to determine the optimal split ratios in the external search loop based on Newton's method, Equation (12).

$$\{y\}_{n+1} = \{y\}_n - \frac{\partial C_{\text{TAC}} / \partial \{y\}_n}{\partial^2 C_{\text{TAC}} / \partial \{y\}_n^{\ 2}} \tag{12}$$

Since the continuous variables obtained from the LP results have been determined without taking into account trade-offs among the cost of utility and the heat transfer areas simultaneously, the resulting TAC cannot be the minimum possible. Consequently, a correction procedure (COP) will be required to improve the network's TAC.

The correction procedure (COP)

The COP aims to modify the heat loads of internal exchangers, as well as split ratios within a reasonable search range. Equation 13 defines this range for the heat load of each internal exchanger:

$$\left[0\ Q_{max}\right] \quad \text{which}$$
$$Q_{max} = \min\left\{W_{H_i}\left(T_{H_i}^{in} - T_{H_i}^{out}\right),\ W_{C_j}\left(T_{C_j}^{out} - T_{C_j}^{in}\right)\right\} \tag{13}$$

For split ratios, the search range remains the same as in the preceding section. The results of the previous LP section are taken into account as initial values for the variables used in the COP section. Following the setting of these variables in the external search loop of this section, the stream's temperatures can be computed by the Equations (2, 3, and 5). Note that these equations become linear after the split ratios and heat load of the internal exchangers are specified by the external search loop.

Next, the ΔT of exchangers is calculated. If ΔT is not positive for at least one exchanger, the values considered for the variables in the search loop will be thermodynamically unreasonable, so other values are chosen for them. Otherwise, the heat loads of utility exchangers located at the end of streams will be calculated using Equation (4). The areas are then computed by Equations (10) and (11), and the TAC for the network is determined according to Equation (9). Based on the calculated TAC, the search loop modifies the heat load of exchangers and split ratios. The procedure continues until the network reaches its lowest TAC.

For small networks, the search operation can be managed by Newton's method, similar to Equation (12). In contrast, stochastic algorithms would be more efficient for large-scale networks. A suitable algorithm for this purpose is the imperialist competition algorithm (ICA), which has demonstrated its ability to address continuous problems, especially in the recent publication by Feyli *et al.*[36]

When the subsections of the methodology section are combined, one can carry out the optimization algorithm shown in Fig. 1. The performance of this relatively linear approach was tested with four medium- to large-scale case studies, described in the next section.

Illustrative examples

Four benchmark examples from the literature were chosen to validate the proposed approach. Programming was done in MATLAB, which contains some of the optimization functions and codes for the process of the GA and decoding of the EAM to form the constraints of the optimization. Despite MATLAB's user-friendliness, it is too slow to handle these problems. Therefore, the original codes were compiled into the C programming language. The final codes were loaded and run on a system consisting of five parallel computers with 6 GB of RAM and a 3.4 GHz processor, and a master processor.

The general characteristics of the case studies are shown in Table 1. For all case studies, as in the work of Feyli *et al.*[4], the rate of the GA operators (i.e., reproduction and crossover) were considered to be 50 %. It should be noted that 5 % of the reproduction operator included the elitism operator. Also, a variable rate based on the maximum and minimum TAC of each generation was used for the mutation operator. The number of genes, *NG*, in each network depends on the size of the problem. The appropriate *NG* has been listed in Table 1 for each case study after examining several numbers. The same process was implemented for the initial population and the number of generations in the GA. According to the number of splits in each gene and the presence of bifurcations in the main streams, the minimum and the maximum number of continuous variables in each GA-produced network are determined.

Based on these explanations, the running time for each case study is stated in the last column of Table 1, which can be reduced by increasing the number of slave computers and improving their characteristics. For the running time of each case study, roughly 5 % is related to the performance of GA in the production of HEN structures, while the share of LP and COP models is approximately 45 % and 50 %, respectively.

In order to make an accurate comparison with references, single-pass countercurrent heat exchangers without phase change were considered in all case studies. Furthermore, the costs associated with piping and connections were not considered, and pressure drop, fouling, and other thermal resistances have been ignored. In the following figures, the thermal load of the exchangers is indicated by underlined numbers.

Case study 1

In this case study, there are six hot and four cold process streams, along with one hot and one cold utility. Several studies have discussed it both with and without taking into account the fixed cap-

Table 1 – *Overview of the presented case studies*

Case Study	*NH*	*NC*	*NG*	Available utilities	Min. continuous variables	Max. continuous variables	Initial population	No. of generations	Running time (min)
1[a]	6	4	8	2	107	171	40	45	81.5
2[a,c]	6	10	10	3	219	299	50	50	152.2
3[b]	13	7	20	2	461	621	60	60	225.4
4[b]	22	17	24	2	1,049	1,241	100	100	613.8

[a]With a maximum of three splits in each gene.

[b]With a maximum of two splits in each gene.

[c]Bifurcation of H_2 stream.

Table 2 – *Stream and cost data for Case study 1*

Stream	T^{in} (°C)	T^{out} (°C)	W (kW K^{-1})	h (kW m^{-2} K^{-1})	Cost (\$ kW^{-1} y^{-1})
H_1	85	45	156.3	0.05	
H_2	120	40	50.0	0.05	
H_3	125	35	23.9	0.05	
H_4	56	46	1250	0.05	
H_5	90	86	1500	0.05	
H_6	225	75	50.0	0.05	
C_1	40	55	466.7	0.05	
C_2	55	65	600	0.05	
C_3	65	165	180	0.05	
C_4	10	170	81.3	0.05	
Steam	200	198		0.05	100
Water	15	20		0.05	15
Annual cost (\$ y^{-1}): 8000+60 A for all HEs (A in m^2)					

Table 3 – *Comparison of the results for Case study 1*

	Units	Q_{HU} (kW)	Q_{CU} (kW)	Total area (m^2)	TAC (\$ y^{-1})
Huang *et al.*[46]	12	20,392.3	14,786.8	56,354.1	5,737,274
Huang and Karimi[47]	11	20,206.6	14,601.1	56,766.7	5,733,679
Wu *et al.*[48]	13	20,439.1	14,833.7	–[a]	5,722,602
Pavão *et al.*[49]	12	–[a]	–[a]	–[a]	5,715,026
Bao *et al.*[21]	14	20,323.7	14,720.0	56,082.9	5,730,140
Rathjens and Fieg[5]	12	20,420.71	14,815.22	–[a]	5,713,267
Chen *et al.*[50]	12	20,335.5	14,732.5	56,053.4	5,713,746
This work, Fig. 4b	13	20,377.78	14,772.68	55,684.99	5,704,467

[a]data not reported by the authors

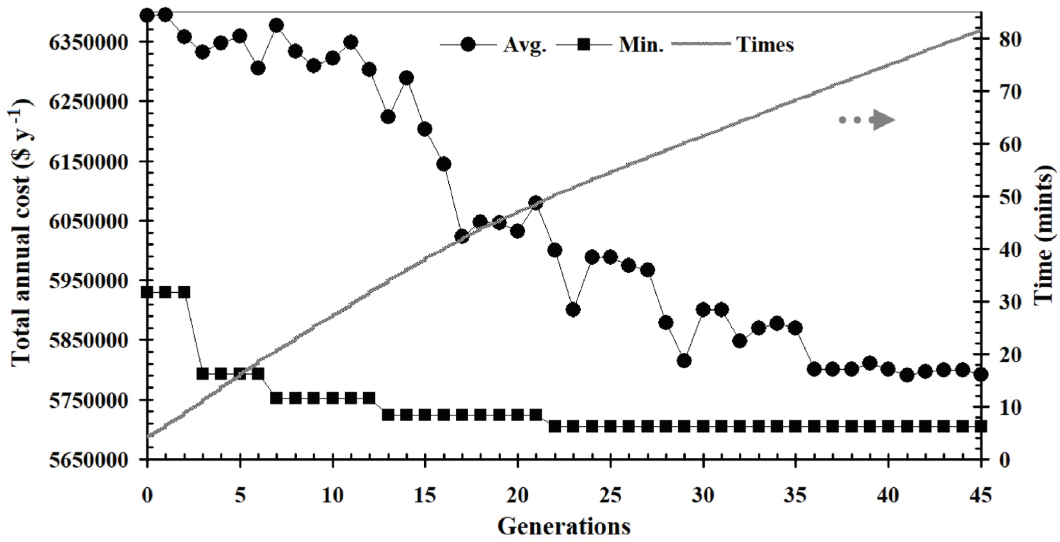
ital cost of exchangers. This paper examines only the case of a fixed capital cost since it is more realistic. The corresponding data are presented in Table 2, taken from Huang *et al.*[46]

The lowest TAC reported in the literature for this example was 5,713,267 \$ y^{-1} obtained by Rathjens and Fieg[5]. They employed a hybrid SIR-HGA approach to tackle the problem with a running time of 3.7 minutes. The optimal network reported in this work, did not take into account internal utilities.

Based on the proposed approach, Fig. 4a illustrates GA performance and the required time to reach the best solution, which is approximately 81.5 minutes. As can be seen from this figure, the downward trend of the best and average TAC curves, as well as the repetition of the best answer adequately over the few end generations, shows the suitable choice of GA parameters. Despite the relatively long running time, the TAC of the final optimal network, as shown in Fig. 4b, is reduced by 8800 \$ $^{y-1}$ compared to Rathjens and Fieg[5]. In the optimal network, an internal hot utility has been utilized on the cold stream, C_3. It is noteworthy that by using this utility at the end of the C_3 stream and retaining the rest of the optimal structure, the TAC of the network would increase to 5,715,853.3 \$ y^{-1}. Fig. 5 illustrates the importance of using internal utility exchangers in this case study. As shown in this figure, the heat load of both HU and E_1 exchangers had not changed. The changes in ΔT around the exchangers led to changes in their heat transfer areas, which affected the TAC directly. The heat transfer areas are indicated in parentheses in Fig. 5.

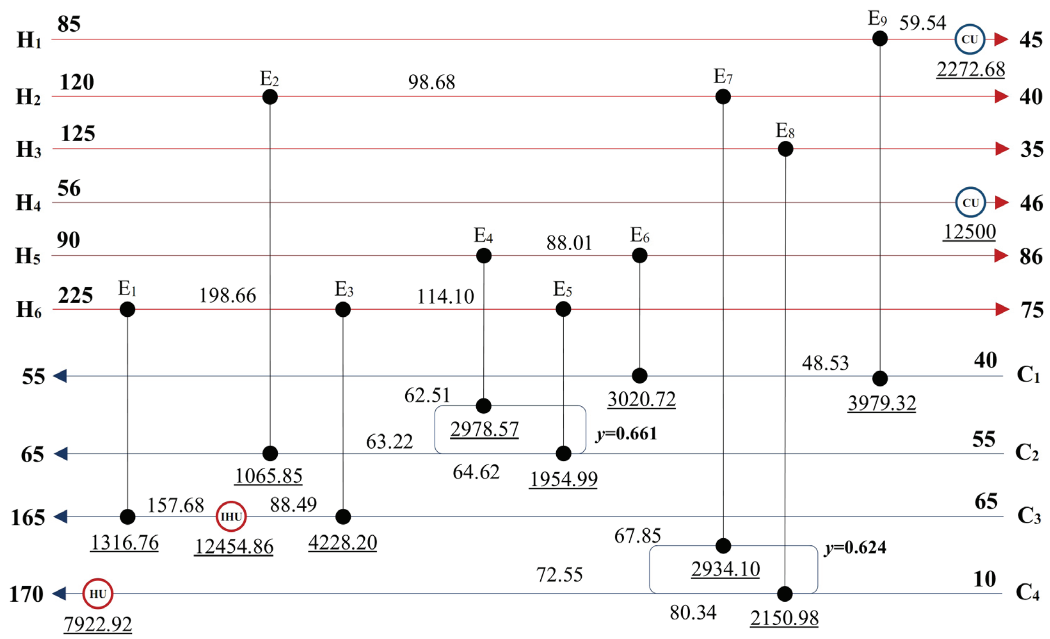
Table 3 provides a comparison of the solutions. Comparison of the TAC obtained in this paper with other approaches suggests that this approach has achieved a TAC of at least 0.154 % lower than other networks reported for this case study.

(a)



Min.: Minimum total annual cost of networks in each generation.
Avg.: Average total annual cost of networks in each generation.

(b)



ΔT_{min} =19.541 °C, TAC =5,704,467.2 \$ y^{-1}

Fig. 4 – *(a) GA performance (b) Final optimal structure; Case study 1*

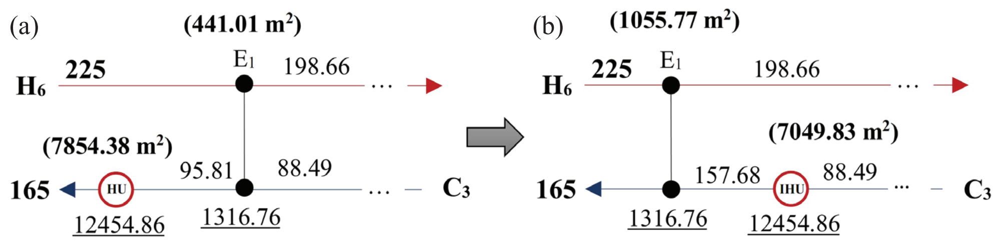


Fig. 5 – *Difference in hot utility placement (a) utility at the stream end (b) inner utility in Fig. 4b*

Case study 2

The second benchmark case includes six hot and ten cold process streams, as well as two hot and one cold utility, taken from Khorasany and Fesanghary[25]. Table 4 presents the relevant data. It is a real-world relatively large-scale network, a simplified dataset of the Bandar Imam Aromatics plant, located on the northeast coast of the Persian Gulf. Because of the high heat capacity flow rate and temperature range of the H_2 stream, bifurcation of this stream upon entering the network was permitted in production structures by GA to achieve adequate energy recovery.

Using the method in this paper, a network, Fig. 6, was obtained with 12 process exchangers, four cold utilities, and two hot utilities, with a TAC of 6,668,380 \$ y^{-1}. According to this figure, only the hot utility HU_1 is utilized in the final structure. Additionally, most connections among hot streams are related to the H_2 stream because of its high potential heat capacity. Another noteworthy point is that no process exchanger is used on streams H_3, H_4, and H_5, as is observed in most of the reported results for this example.

One of the distinctive features of GA is that it yields not only one optimal solution. The second-best structure, with a TAC of 6,672,542.32 \$ y^{-1}, can be seen in Fig. 7. There are many similarities between the second optimal structure and the final one. However, the second optimal structure consumes slightly more hot and cold utilities, and has one additional process exchanger and split.

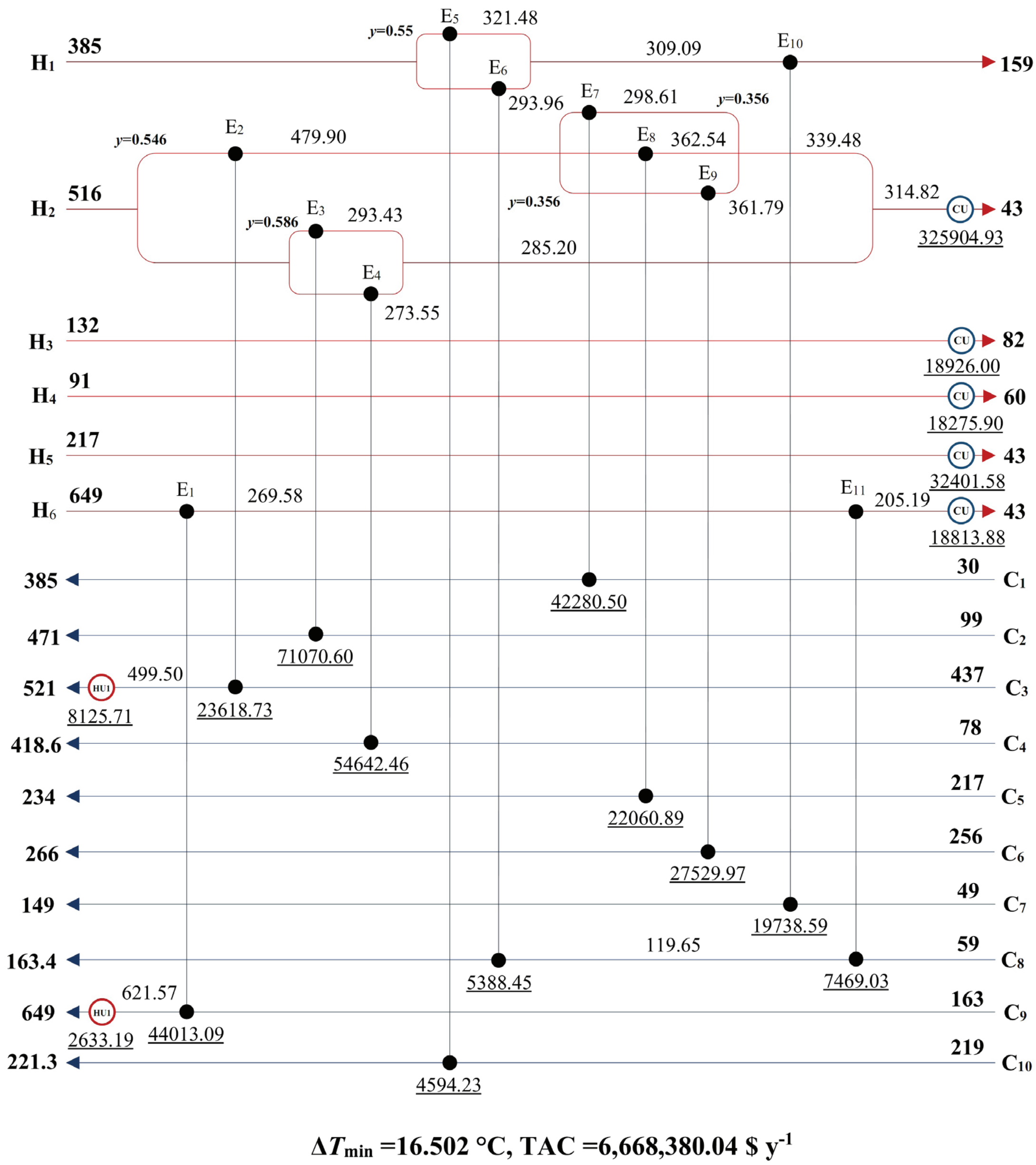


Fig. 6 – *Optimal HEN for Case study 2*

Table 4 – *Stream and cost data for Case study 2*

Stream	T^{in} (°C)	T^{out} (°C)	W (kW K^{-1})	h (kW m^{-2} K^{-1})	Cost ($ kW^{-1} y^{-1})
H_1	385	159	131.51	1.238	
H_2	516	43	1198.96	0.546	
H_3	132	82	378.52	0.771	
H_4	91	60	589.545	0.859	
H_5	217	43	186.216	1	
H_6	649	43	116	1	
C_1	30	385	119.1	1.85	
C_2	99	471	191.05	1.129	
C_3	437	521	377.91	0.815	
C_4	78	418.6	160.43	1	
C_5	217	234	1297.7	0.443	
C_6	256	266	2753	2.085	
C_7	49	149	197.39	1	
C_8	59	163.4	123.156	1.063	
C_9	163	649	95.98	1.81	
C_{10}	219	221.3	1997.5	1.377	
HU1	1800	800		1.2	35
HU2	509	509		1.0	27
CU	38	82		1.0	2.1
Annual cost ($ y^{-1}): 26600+4147.5 $A^{0.6}$ for all HEs (A in m^2)					

Table 5 – *Comparison of the results for Case study 2*

	Units	Q_{HU} (kW)	Q_{CU} (kW)	Total area (m^2)	TAC ($ y^{-1})
Existing plant	18	122.16	524.72	–[a]	8,856,412
Khorasany and Fesanghary[25]	18	66.07	469.62	–[a]	7,435,740
Zhaoyi *et al.*[38]	16	38.80	442.37	–[a]	7,361,190
Pavão *et al.*[51]	17	34.21	437.78	30,897.2	7,301,437
Zhang and Cui[33]	18	10.63	414.19	31,397.0	6,861,111
Bao *et al.*[21]	19	10.05	413.61	–[a]	6,869,610
Pavão *et al.*[40]	18	9.50	413.07	31,012.2	6,712,551
Xiao *et al.*[52]	19	9.93	413.50	–[a]	6,798,067
Xiao *et al.*[53]	18	10.444	414.014	–[a]	6,692,513
Kayange *et al.*[6]	18	9.62	413.18	31,076.3	6,716,343
Rathjens and Fieg[5]	18	9.710	413.27	–[a]	6,657,080
Xiao *et al.*[54]	19	9.911	413.373	–[a]	6,718,533
Xu *et al.*[45]	18	9.65	413.23	30,800.7	6,652,137
Feily *et al.*[4]	17	34.205	437.765	–[a]	7,128,522
Liu *et al.*[55]	18	9.548	413.110	30,820	6,664,749
This work, Fig. 6	18	10.759	414.322	30,627.28	6,668,380
This work, Fig. 7	19	10.801	414.363	30,617.88	6,672,542

[a] data not reported by the authors

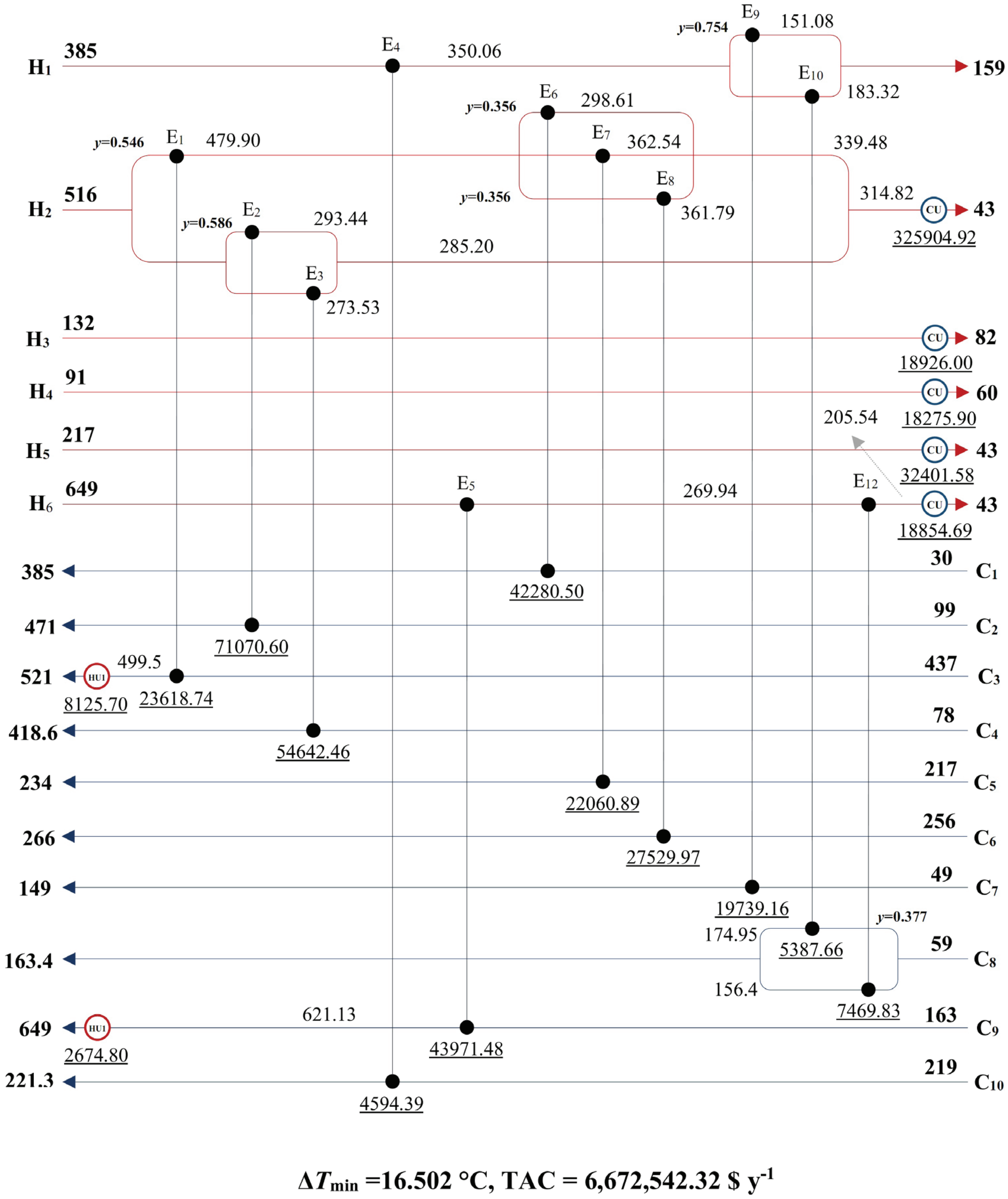


Fig. 7 – *Second-best network for Case study 2*

Table 5 provides a comparison with obtained results in the literature. The best solution reported in the literature belongs to Rathjens and Fieg[5] and Xu *et al.*[45] Using a tailored hybrid approach (SIRHGA), Rathjens and Fieg[5] achieved a TAC of 6,657,080 $ y^{-1} after 248.4 minutes, approximately 11,300 $ y^{-1} less than this paper's solution. On the other hand, Xu *et al.*[45], employing an optimization route arrangement approach incorporating several modules based on RWCE, obtained 6,652,137 $ y^{-1} for the network's TAC, with a running time of 79.5 hours. Compared with this paper's result, this solution is 16,243 $ y^{-1} less, and also represents the lowest TAC in the literature for this case study. In addition to the efficient strategy for optimizing continuous variables in Rathjens and Fieg[5] and Xu *et al.*[45], the main reason for the better solutions was the possibility of up to five splits on the H_2 stream. That feature provided better recovery of H_2 stream's energy at a lower annual cost.

However, despite its simplicity and the possibility of considering up to three splits on streams, the approach proposed herein has been able to obtain an acceptable TAC for this network in a reasonable computational time. Implementing the optimal network obtained in this paper resulted in economic savings of 2.184 M$ y^{-1} through heat integration compared to the existing plant.

Case study 3

The third case study involves 20 process streams (13 hot and 7 cold streams) along with a single hot and cold utility, provided originally by Escobar and Trierweiler[56]. Numerous researchers have taken an interest in this case. Table 6 gives stream data and cost information.

Based on the literature, the best optimal solution for this case has been reported by Zamora *et al.*[43] with a TAC of 1,388,812 $ y^{-1}. A two-stage approach was used by the authors to optimize and relocate utility exchangers considering a base HEN design, which was adapted from the optimal network of Xiao *et al.*[52]

The best optimal network obtained utilizing the presented novel procedure for this case study is shown in Fig. 8, which includes 18 process exchangers and three hot utility exchangers with a TAC of 1,388,609.47 $ y^{-1}, approximately 202 $ y^{-1} less than that of Zamora *et al.*[43] One of the features of this optimal structure is how to implement the hot utility on the C_7 stream. It has been very effective at reducing the network's TAC.

Furthermore, Fig. 9 illustrates the second structure obtained with a TAC of 1,403,226.97 $ y^{-1}. Although the number of exchangers is the same in these two structures, due to the arrangement of exchangers and the effective use of internal utility in the best optimal network, Fig. 8, there is a difference of 14,617.5 $ y^{-1} in their TAC.

The comparison of reported TACs by various methods in the literature is presented in Table 7, which validates the effectiveness of the proposed approach in dealing with large-scale HEN synthesis problems.

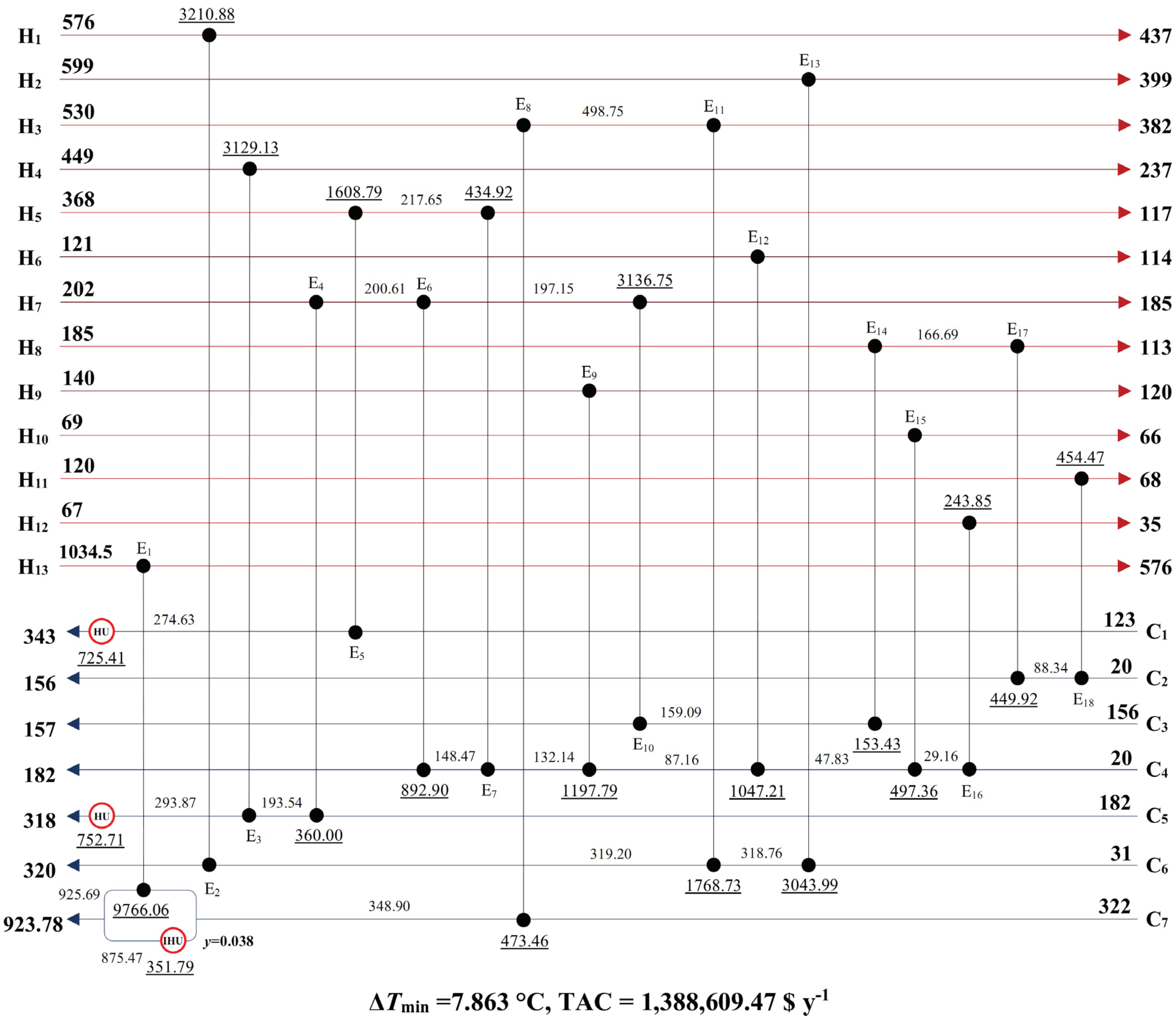


Fig. 8 – *Optimal HEN for Case study 3*

Table 6 – *Stream and cost data for Case Study 3*

Stream	T^{in} (°C)	T^{out} (°C)	W (kW K^{-1})	h (kW m^{-2} K^{-1})	Cost (\$ kW^{-1} y^{-1})
H_1	576	437	23.10	0.06	
H_2	599	399	15.22	0.06	
H_3	530	282	15.15	0.06	
H_4	449	237	14.76	0.06	
H_5	368	117	10.70	0.06	
H_6	121	114	149.60	1	
H_7	202	185	258.20	1	
H_8	185	113	8.38	1	
H_9	140	120	59.89	1	
H_{10}	69	66	165.79	1	
H_{11}	120	68	8.74	1	
H_{12}	67	35	7.62	1	
H_{13}	1034.5	576	21.30	0.06	
C_1	123	343	10.61	0.06	
C_2	20	156	6.65	1.20	
C_3	156	157	3291.00	2.00	
C_4	20	182	26.63	1.20	
C_5	182	318	31.19	1.20	
C_6	318	320	4011.83	2.00	
C_7	322	923.78	17.6	0.06	
HU	927	927		5.0	250
CU	9	17		1.0	25
Annual cost (\$ y^{-1}): 4000+500 $A^{0.83}$ for all HEs (A in m^2)					

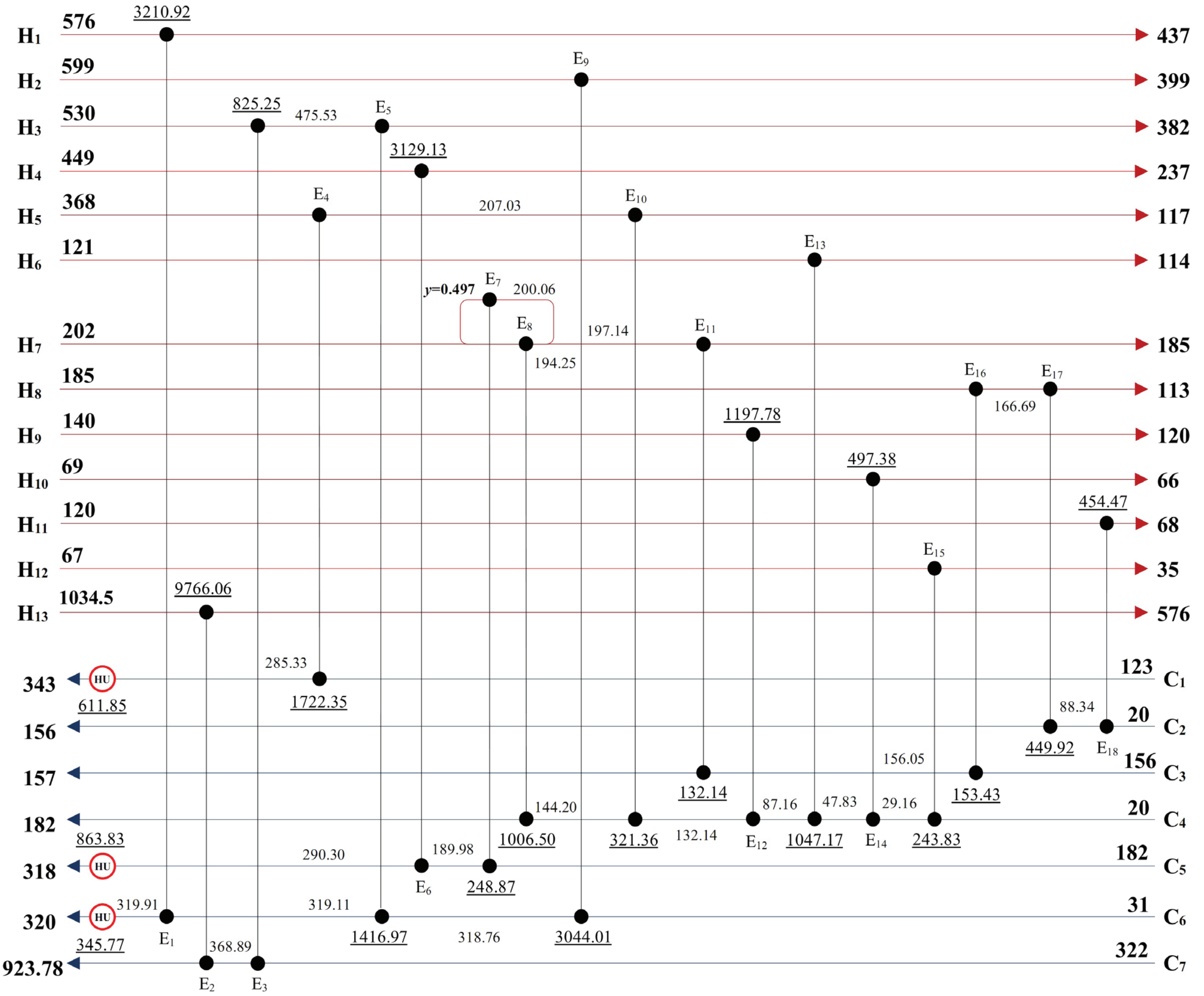
Table 7 – *Comparison of the results for Case study 3*

	Units	Q_{HU} (kW)	Q_{CU} (kW)	Total area (m^2)	TAC (\$ y^{-1})
Escobar and Trierweiler[56]	21	1937.998	106.93	5425.41	1,521,158[b]
Xiao and Cui[35]	23	2198.34	367.28	5077.79	1,549,795[a]
Pavão *et al.*[40]	22	2074.908	243.84	5038.76	1,467,805[b]
Bao *et al.*[21]	22	2077.51	250.435	5053.20	1,462,323
Zhang and Cui[33]	22	1831.07	0.00	5110.00	1,418,981
Xu *et al.*[57]	21	1830.8	36.6	–[c]	1,412,801
Xiao *et al.*[52]	21	1831.068	0.00	5091.00	1,403,581
Xiao *et al.*[58]	20	1831.00	0.00	–[c]	1,395,587
Rathjens and Fieg[5]	20	1831.07	0.00	–[c]	1,407,203
Zamora *et al.*[43]	21	1831.068	0.00	4938.23	1,388,812
Xiao *et al.*[54]	21	1831.10	0.00	–[c]	1,396,471
This work, Fig. 8	21	1829.91	0.00	4938.87	1,388,609
This work, Fig. 9	21	1827.445	0.00	5093.98	1,403,227

[a]Revised by Zhang and Cui[33]

[b]Revised by Zamora *et al.*[43]

[c]data not reported by the authors



ΔT_{min} =7.865 °C, TAC = 1,403,226.97 $ y^{-1}

Fig. 9 – *Second-best network for Case study 3*

Case study 4

This case, known as a large-scale HEN, includes 39 process streams (22 hot and 17 cold streams) along with a single hot and cold utility. Table 8 presents the corresponding data, which have been adapted from Björk and Pettersson[59] and have also been investigated by numerous researchers.

The best results in the literature for this case study are reported by Rathjens and Fieg[5] and Xu *et al.*[45] Rathjens and Fieg[5] achieved a TAC of 1,852,913 $ y^{-1} utilizing a combined HGA-SIR approach with a running time of approximately 6.62 hours. The TAC of the final network of Xu *et al.*[45], employing the route arrangement optimization strategy, was 1,852,723.1 $ y^{-1}, with a running time of 138 hours. It was only 190 $ y^{-1} better than that of Rathjens and Fieg[5].

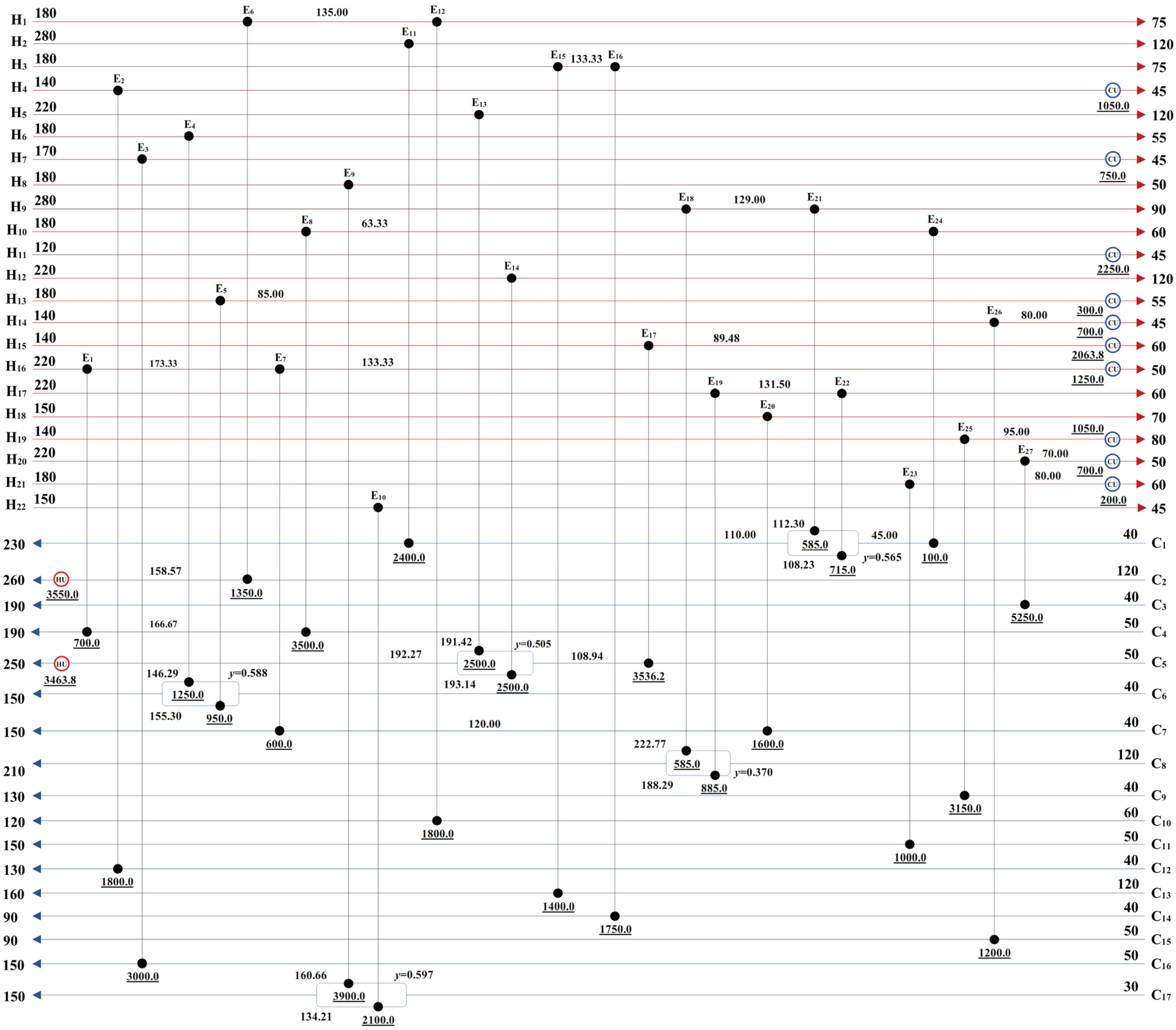
The solution obtained by the proposed approach led to the lowest TAC of 1,840,935.1 $ y^{-1} with a running time of 613.8 minutes. The optimal structure is illustrated in Fig. 10, consisting of 27 process exchangers, ten cold utility, and two hot utility exchangers along with four stream splitting.

A comparison with the results for the Case study 4 is presented in Table 9. The obtained results in this paper indicate a decrease of 11,788 $ y^{-1} (0.636 %) in the TAC of the network compared to the best results reported in the literature[5,45]. Another significant point is its reasonable operating time, especially compared to Xu *et al.*[45], since the proposed approach is much more straightforward than the method of Xu *et al.*[45] These results confirm the high performance of the proposed approach in dealing with large-scale HENs compared to others.

Fig. 11 illustrates the performance of the proposed approach with and without considering the correction procedure (COP) for this case study. As it is clear from this figure, without employing COP,

Table 8 – *Stream and cost data for Case study 4*

Stream	T^{in} (°C)	T^{out} (°C)	W (kW K^{-1})	h (kW m^{-2} K^{-1})	Cost (\$ kW^{-1} y^{-1})
H_1	180	75	30	2	
H_2	280	120	15	2.5	
H_3	180	75	30	2	
H_4	140	45	30	2	
H_5	220	120	25	1.5	
H_6	180	55	10	2	
H_7	170	45	30	2	
H_8	180	50	30	2	
H_9	280	90	15	2	
H_{10}	180	60	30	2	
H_{11}	120	45	30	2	
H_{12}	220	120	25	2	
H_{13}	180	55	10	2	
H_{14}	140	45	20	2	
H_{15}	140	60	70	2	
H_{16}	220	50	15	2.5	
H_{17}	220	60	10	2.5	
H_{18}	150	70	20	2	
H_{19}	140	80	70	2	
H_{20}	220	50	35	2	
H_{21}	180	60	10	2	
H_{22}	150	45	20	2.5	
C_1	40	230	20	1.5	
C_2	120	260	35	1	
C_3	40	190	35	1.5	
C_4	50	190	30	2	
C_5	50	250	60	2	
C_6	40	150	20	2	
C_7	40	150	20	2	
C_8	120	210	35	2.5	
C_9	40	130	35	2.5	
C_{10}	60	120	30	2.5	
C_{11}	50	150	10	3	
C_{12}	40	130	20	1	
C_{13}	120	160	35	1	
C_{14}	40	90	35	1.75	
C_{15}	50	90	30	1.5	
C_{16}	50	150	30	2	
C_{17}	30	150	50	2	
HU	325	325		1	70
CU	25	40		2	10
Annual cost (\$ y^{-1}): 8000+800 $A^{0.8}$ for all HEs (A in m^2)					



ΔT_{min} =6.667 °C, TAC = 1,840,935.1 \$ y^{-1}

Fig. 10 – *Optimal HEN for Case study 4*

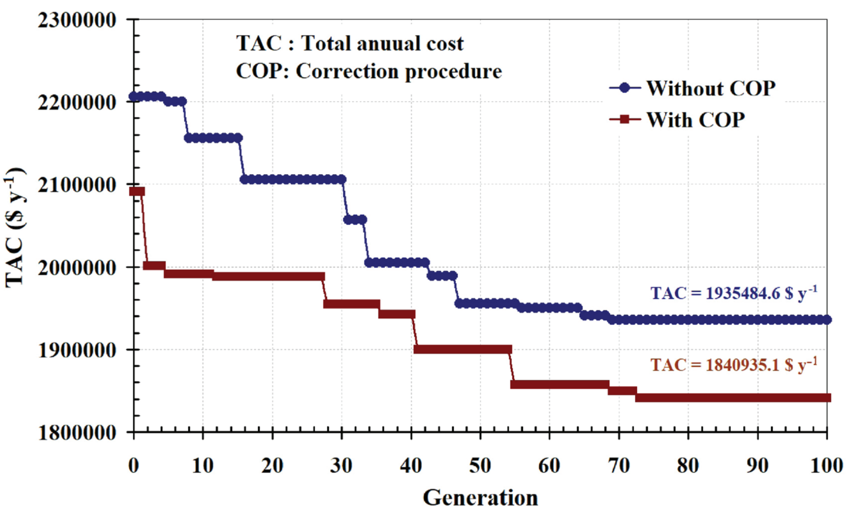


Fig. 11 – *Comparison of the performance of the proposed approach with and without utilizing correction procedure (COP) in Case study 4*

Table 9 – *Comparison of the results for Case study 4*

	Units	Q_{HU} (kW)	Q_{CU} (kW)	Total area (m^2)	TAC (\$ y^{-1})
Björk and Pettersson[59]	48	10,550	13,850	–[b]	2,073,251
Escobar and Trierweiler[56]	44	6,226	9,526	3,992	2,055,421
Huang and Karimi[47]	41	7,914	11,214	3,027	1,961,086
Pavão *et al.*[49]	42	8,184[a]	11,484[a]	2,636[a]	1,900,614
Nemet *et al.*[60]	44	7,350	10,650	2,881	1,928,800
Xiao *et al.*[52]	45	8,336	11,636	2,649	1,925,783
Xiao *et al.*[53]	44	7,745	11,045	–[b]	1,873,813
Xiao *et al.*[58]	43	8,351	11,591	–[b]	1,921,639
Rathjens and Fieg[5]	41	7,210.12	10,510.12	–[b]	1,852,913
Zhang *et al.*[61]	42	8,009	11,309	2,730	1,918,593
Xiao *et al.*[54]	43	7,994.7	11,294.7	–[b]	1,910,630
Xu *et al.*[45]	41	6,897.48	10,197.5	–[b]	1,852,723.1
This work, Fig. 10	39	7,013.82	10,313.82	2,911.58	1,840,935.1

[a]Calculated by Zhang *et al.*[61]

[b]data not reported by the authors

the TAC of this case study reaches 1,935,484.6 \$ y^{-1}, which is about 4.9 % more than when COP was utilized in the proposed approach.

Conclusion

In this article, an effective approach for optimizing HENs is proposed based on the GA-LP-COP methods with two external search loops. A modified node representation model with stream splitting and multiple locations for utility exchangers is used to create the configuration of the networks by GA. The benefit of this type of network representation is that only feasible HEN structures are produced during the optimization process. Based on MER, LP optimizes continuous variables. A linear correction procedure (COP) is then applied to reach the lowest TAC of the networks. The significance of using COP in the proposed approach is shown in the Case study 4, Fig. 11, in increasing the likelihood of obtaining better solutions. Despite the simplicity of the proposed approach, the case studies demonstrate that the novel method often finds better results than those reported in the literature.

One of the advantages of this approach is that it requires no initialization due to the use of GA in structural optimization, the simplex method in the LP formulation, and a linear procedure in the COP section. The only variables that require initial guesses are the split ratios and internal exchangers' heat loads in the LP and COP sections' search loops.

In this novel approach, to control the superstructure and search space, as well as to reduce the optimization time, at most three branches were allowed in the splitters within the genes and bifurcation of some potential streams upon entering the network. Therefore, the application of the current approach to large-scale industrial networks, such as the preheating of crude oil in refineries, may require the addition of branches. The number of branches, however, can be adjusted according to some heuristics suggested by Lee and Hua[62], which were based on the heat capacity flow rates of streams or the existing HEN structure.

Nomenclature

A	– heat transfer area of exchanger, m^2
AF	– annualization factor
C_f	– fixed capital cost
C_A	– area cost coefficient
C_{CU} and C_{HU}	– cold and hot utility cost coefficients, \$ kW^{-1} y^{-1}
h	– heat transfer coefficient, kW m^{-2} K^{-1}
I	– interest rate, %
$LMTD$	– logarithmic mean temperature difference, °C
n	– plant lifetime
NC	– number of cold process streams
NG	– number of genes of a HEN
NH	– number of hot process streams
NIE	– number of internal exchangers in a network
NPE	– number of process exchangers in a network

Q	–	heat load of an exchanger, kW
Q_{max}	–	maximum possible heat load of an internal exchanger, kW
T	–	stream temperature, °C
TAC	–	total annual cost, \$ y^{-1}
ΔT	–	approach temperature difference in hot or cold end of exchangers, °C
ΔT_{min}	–	minimum approach temperature difference, °C
U	–	overall heat transfer coefficient, kW m^{-2} K^{-1}
W	–	heat capacity flow rate, kW K^{-1}
y	–	split ratio of streams
β	–	exponent for area

Indices

C	–	cold stream
CU	–	cold utility
g	–	gene index
H	–	hot stream
HU	–	hot utility
i	–	hot stream index
j	–	cold stream index
k	–	internal exchanger index
in	–	inlet
out	–	outlet

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