Scale-up of Processes Using Material Systems with Variable Physical Properties*

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In memoriam Prof. Emeritus Vera Johanides

According to the theory of similarity, two processes are similar to one another if they take place in a similar geometrical space, and if all the dimensionless numbers necessary to describe the process, have the same numerical value. A complete similarity requires a geometrical, material and process-related similarity.

This contribution aims at problems concerning material similarity. In many industrial tasks this problem presents no difficulties. In order to find an alternative fluid for model measurements, the model experiments are performed with Newtonian fluids of different viscosities to achieve the flow range in question on industrial scale, or one sticks to the same material system because it cannot be specified physically (slurries, slimes, foams, etc.)

A problem arises when model (laboratory, bench-scale) measurements are to be performed in a so-called "cold model", but the industrial plant operates at high temperatures (petrochemicals; $T \approx 800 - 1000$ °C). How can we ascertain that the laboratory model system behaves hydrodynamically similarly to that in the industrial plant? Here, different temperature dependence of physical properties (viscosity, density) can cause problems.

A problem arises when laboratory measurements are to be performed with cheap and problem-free model fluids to gain information about scaling-up of an apparatus for the treatment of cell cultures in biotechnology (mammal and plant cells, aerobic cultures, yeasts), the rheological behaviour of which is very complex (non-Newtonian: pseudoplactic and viscoelastic). Which model system may we choose?

These are only two of the many problems which will be addressed and treated in this paper. The answer will be clear and unambiguous: We may choose any model material system whose *dimensionless material function* in question is similar to that of the original material system. It will be shown how to proceed to arrive at it.

Keywords:
Dimensionless material functions, variable physical properties, rheological properties

Treatment of temperature dependent physical properties

Introductional remarks

When using the dimensional analysis to tackle engineering problems, it is generally assumed that physical properties of the material system remain unaltered in the course of the process. Relationships such as the heat transfer characteristic of an industrial device (e.g., vessel, pipe):

$$Nu = f\,(Re,\ Pr) \tag{1}$$

are valid for any material system with Newtonian viscosity and for any constant process temperature, i.e. for *constant* physical properties at a given system temperature.

However, constance of physical properties cannot be assumed in every physical process. A temperature *field* may well generate a viscosity *field* or even a density *field* in the material system treated. In non-Newtonian (pseudoplastic or viscoelastic) liquids, a shear rate can also produce a viscosity *field*.

Although most physical properties (e.g. viscosity, density, diffusivity) must be regarded as variables, it is particularly the value of viscosity that can be varied by many orders of magnitude under certain process conditions (Fig. 1). In the following, the dimensional analysis will be applied as an example to describe the variability of this physical property. However, this approach can be adapted for any other physical property.

Dimensionless representation of the material function

Similar behavior of a certain physical property, common to different material systems, can only be

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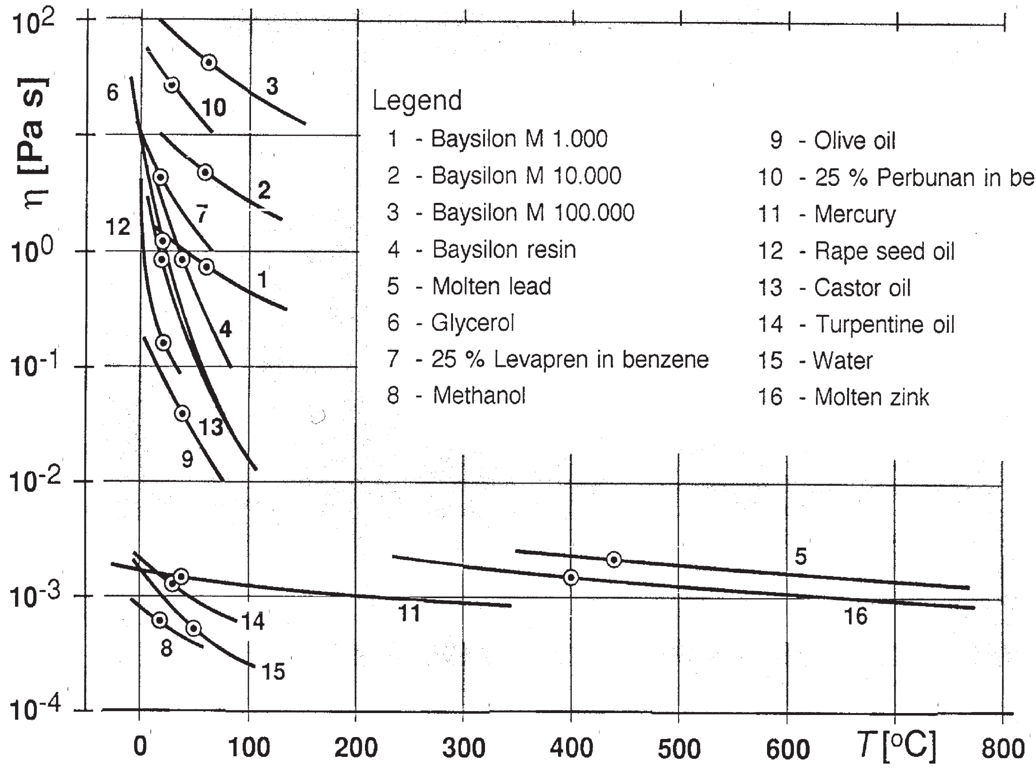


Fig. 1 – *$\eta(T)$ behaviour of same industrially interesting fluids. Some of them show a very feeble temperature dependence of viscosity, whereas others are extremely temperature sensitive*

visualized by a *dimensionless representation* of the material function of that property. It is furthermore desirable to formulate this function as uniformly as possible.

This can be achieved by the "standard representation" of the material function in which a standardizing transformation of the material function $\eta(T)$ is defined in such a way that the expression produced

$$\eta/\eta_0 = f\,\{-\,\gamma_0\,(T - T_0)\} \tag{2}$$

meets the requirement $f\,(0) = f\,'(0) = 1$

$\gamma_0 \equiv \left(\dfrac{1}{\eta_0}\dfrac{\mathrm{d}\eta}{\mathrm{d}T}\right)_{T_0}$ is the temperature coefficient of viscosity and $\eta_0 \equiv \eta(T_0)$ is *any* reference temperature.

Fig. 2 depicts the dramatic effect of this *standard transformation*. It shows that all liquids presented in this diagram behave similarly to one another with respect to $\eta(T)$. This is the more surprising in view of the fact that their viscosities cover a range of 5 orders of magnitude, and also their temperature coefficients of the viscosity γ_0 differ widely.

In Fig. 2 T_0 stands for the mean process temperature. In fact, any temperature could have been taken in the case of these substances as the reference temperature in the range covered by experiments. The presented evaluation of $\eta(T)$ is independent of the reference point T_0, it is *invariant* of it.

Engineers prefer the representation

$$\eta/\eta_0 = \exp\,[-\,\gamma_0\,(T - T_0)] \tag{3}$$

which is a special case of (2), although this is not the best possible approximation (see the dotted line in Fig. 2). Due to this relation (3), the engineering literature prefers the incorporation of the quotient η/η_0 , e.g. η_w/η (w – wall) instead of $\gamma_0\Delta T$ in the process characteristics of heat transfer. The heat transfer characteristics normally read

$$Nu = f\,(Re,\,Pr,\,\eta_w/\eta) \tag{4}$$

Were the function $\eta(T)$ not independent (invariant) of the reference point T_0, an additional number – $\Delta T/T_0$ – had necesarily be incorporated in (4) to fully describe the $\eta(T)$ behaviour.

A slightly better approximation of the material function $\eta(T)$ as compared to the eq. (3) is provided by the well known *Arrhenius* relationship (Fig. 3):

$$\frac{\eta}{\eta_0} = \exp\left[-\frac{E_0}{RT_0}\left(\frac{T_0}{T} - 1\right)\right] \tag{5}$$

This type of representation was first used by *Svante Arrhenius* (1859 – 1927) to describe temperature dependence of the chemical reaction rate con-

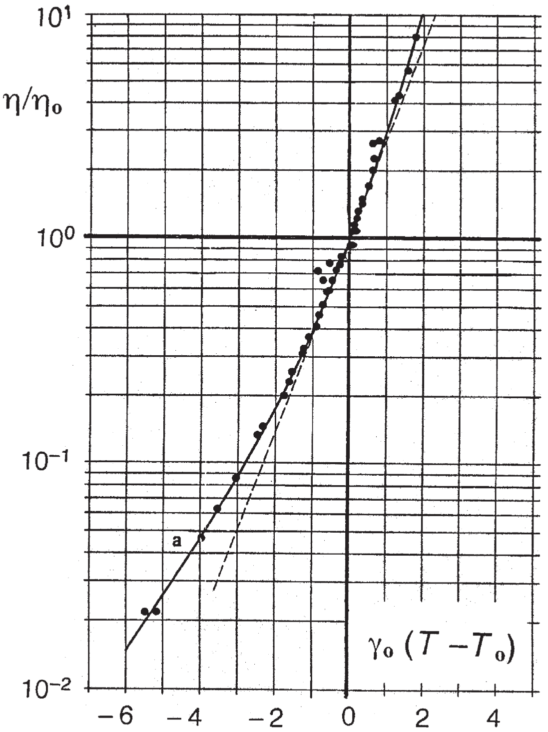


Fig. 2 – *Temperature dependence of viscosity of different fluids and their standard representation, which leads to the reference-invariant approximation of the material function*[1]

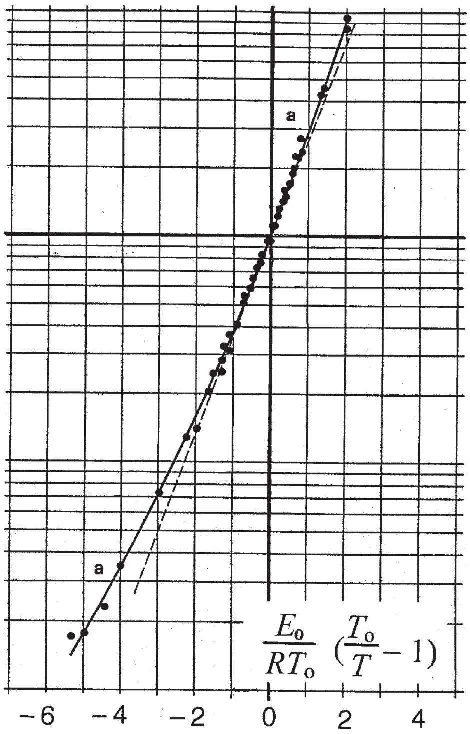


Fig. 3 – *Temperature dependence of fluids from Fig. 1, represented by the standard representation according to Arrhenius*[1]

stant: $k = k_{\infty}$ exp $[E/RT]$, where E [kJ] is the activation energy of the reaction rate constant.

Fig. 4 shows the dependence $\rho(T)$ for some organic fluids and for water and Fig. 5 presents the *standard transformation* of this dependence, where

$$\beta_0 \equiv \left(\frac{1}{\rho_0}\frac{\mathrm{d}\rho}{\mathrm{d}T}\right)_{T_0} \qquad (6)$$

stands for the temperature coefficient of the density.

Now it can be seen that the standard transformation of $\rho(T)$ is *invariant* of the reference point T_0 only for organic fluids considered, but not for water (see the signs o and •). This indicates that water cannot be taken as a model fluid in the so-called "cold model", in order to obtain a reliable information about the behaviour of organic fluids at high temperatures in the industrial plant. Only in the range close to the reference point $-\beta_0\Delta T \approx 0$ – water behaves with respect to $\rho(T)$ similarly to these organic fluids.

Fig. 6 shows a big impact of temperature dependence on the diffusion coefficient $D(T)$ in the system polyamide (Perlon®)/water in the course of the drying of this synthetic fibre. Only when $D(T)$ was taken into account (solid line), the calculation of the drying progress corresponded to the measured course of drying (— • —).

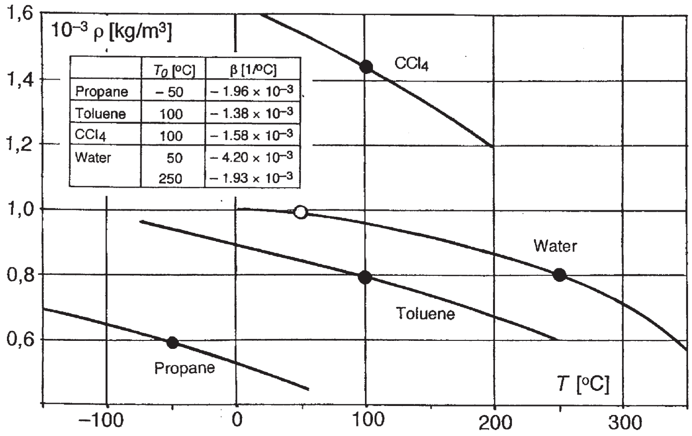


Fig. 4 – *Temperature dependence of fluid density ρ(T) for some organic fluids and for water*[2]

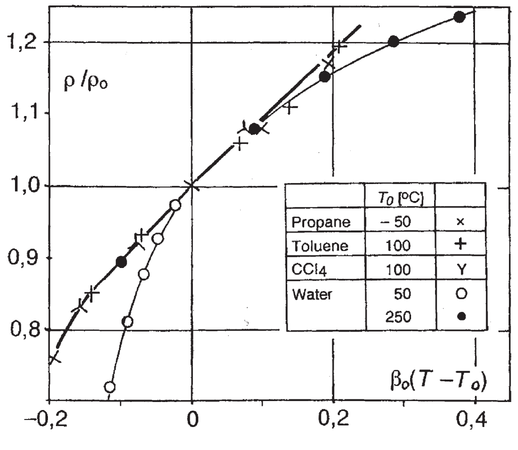


Fig. 5 – *Standard representation ρ(T) for fluids in Fig. 4*[2]

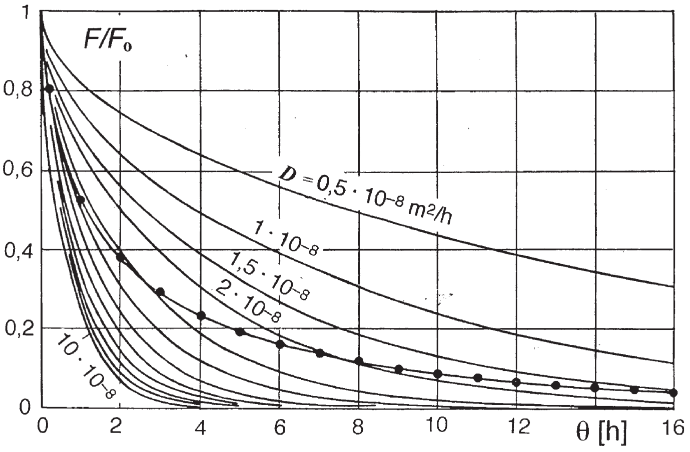


Fig. 6 – *Time course of the relative humidity F/F_0. The impact of the temperature dependence of diffusion coefficient D(T) for polyamide (Perlon®)/water on drying*[3] *(F_0 – humidity at start)*

Treatment of non-Newtonian viscosity behaviour using the dimensional analysis

The main characteristic of Newtonian liquids is that simple shear flow (e.g., Couette flow) generates shear stress τ, which is proportional to the shear rate $\dot{\gamma}$. The proportionality constant, the dynamic viscosity η, is the only material constant in the Newton's Law of flow:

$$\tau = \eta \dot{\gamma} \rightarrow \eta = \tau / \dot{\gamma} \tag{7}$$

η depends only on temperature.

In case of non-Newtonian liquids, η depends on the shear rate $\dot{\gamma}$ as well. These liquids can be classified into various categories of materials depending on their flow behaviour. Here, only pseudoplastic and viscoelastic fluids will be treated.

Pseudoplastic fluids

Pseudoplastic or shear-thinning fluids form a class to which industrially important macromolecular fluids belong. Typical flow behaviour is shown in Fig. 7. The viscosity approaches a constant value at low shear rates called the *zero-shear viscosity* η_0 and a constant value in the limit of the very high shear rates called the *infinite shear viscosity* η_∞. Between these two extremes, shear stress (or the viscosity) is often linear with the shear rate over several decades on logarithmic coordinates. In this range the so-called "power law" or Ostwald – de Waele viscosity is valid. This industrially important viscosity range can be described dimensionlesly by the introduction of two additional material constants, namely η_0 and $\dot{\gamma}_0$, see Fig. 8. Then the material function of this fluid class is given by

$$\{\eta/\eta_0,\ \dot{\gamma}/\dot{\gamma}_0,\ \mathrm{m}\} \tag{8}$$

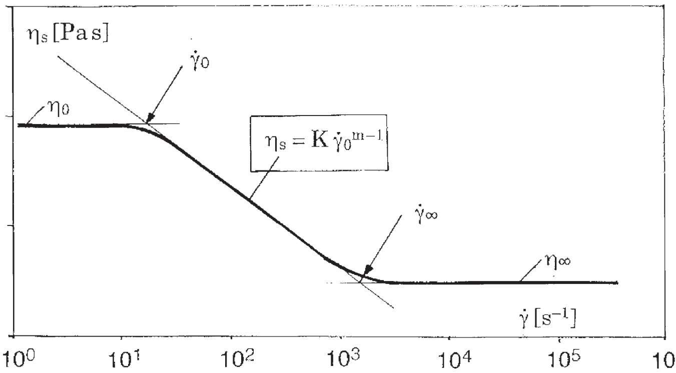
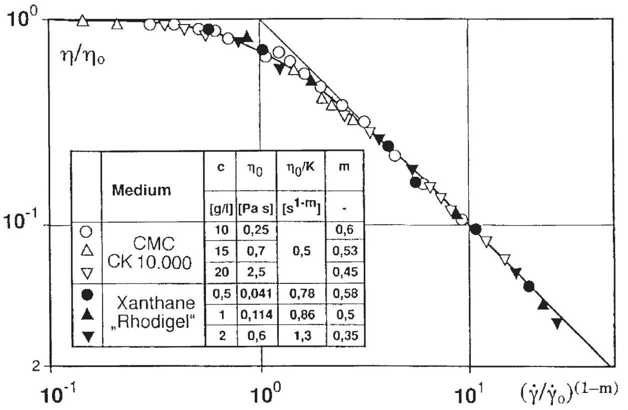


Fig. 7 – *Typical flow behaviour of pseudoplastic fluids*



	Medium	c	η_0	η_0/K	m
		[g/l]	[Pa s]	[s^{1-m}]	-
○	CMC CK 10.000	10	0,25	0,5	0,6
△		15	0,7		0,53
▽		20	2,5		0,45
●	Xanthane „Rhodigel“	0,5	0,041	0,78	0,58
▲		1	0,114	0,86	0,5
▼		2	0,6	1,3	0,35

Fig. 8 – *Dimensionless, standardized material function of some pseudoplastic fluids used as model substances in biotechnological research*[4]

Fig. 8 depicts the dimensionless, standardized material function of some biotechnologically important pseudoplastic fluids. It represents the dimensionless framework for this class of fluids. All fluids of this viscosity class are similar to one another with respect to $\eta(\dot{\gamma})$, if they can be correlated in this framework.

Viscoelastic fluids

Almost every biological solution of low viscosity displays not only viscous but also viscoelastic flow behavior. These liquids are capable of storing part of the deformation energy elastically and reversibly. They evade mechanical stress by contracting like rubber bands. This behavior causes secondary flow, which often runs contrary to the flow produced by mass forces.

Elastic behaviour of liquids is characterized mainly by the ratio of first differences in normal stress, N_1, to the shear stress, τ. This ratio, the Weissenberg number $Wi \equiv N_1/\tau$, is usually represented as a function of the rate of shear $\dot{\gamma}$, see Fig. 9.

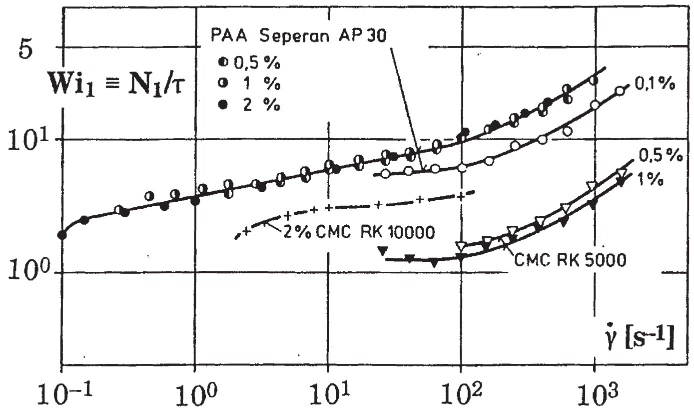
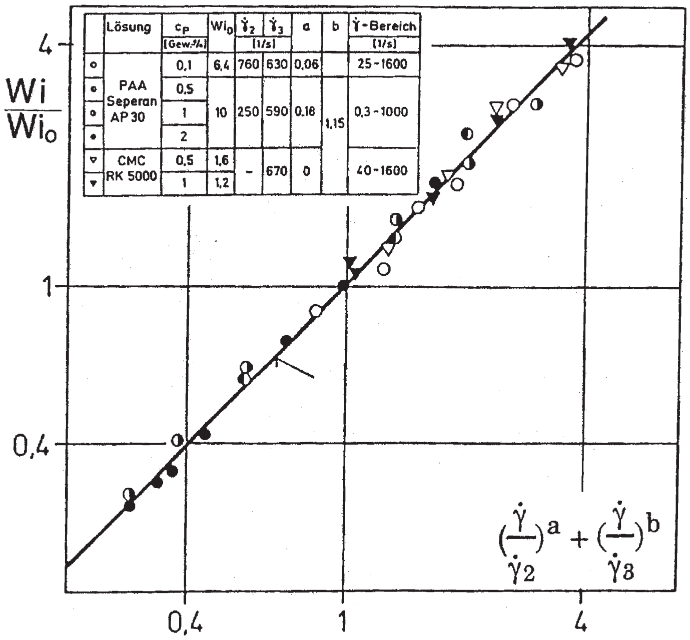


Fig. 9 – *We($\dot{\gamma}$) dependence for some important viscoelastic fluids, used as model substances in biotechnological research*[4]

To transform these material functions into dimensionless forms, a reference Weissenberg number Wi_0, is chosen. This leads to the generation of the characteristic rates of shear $\dot{\gamma}_2$ and $\dot{\gamma}_3$. It thus follows that:

$$Wi\,/\,Wi_0 = (\dot{\gamma}/\dot{\gamma}_2)^a + (\dot{\gamma}/\dot{\gamma}_3)^b \qquad (9)$$

In Fig. 10 the $Wi_1(\dot{\gamma})$ behaviour of the fluids presented in Fig. 9 is depicted as a dimensionless, standardized material function, which confirms that these fluids behave similarly to one another in this context. When biological broths behave in the same way, then the represented fluids can be used as model fluids in the model experiments instead of them.



	Lösung	c_P [Gew.-%]	Wi_0	$\dot{\gamma}_2$ [1/s]	$\dot{\gamma}_3$ [1/s]	a	b	$\dot{\gamma}$-Bereich [1/s]
○	PAA Seperan AP 30	0,1	6,4	760	630	0,06		25 - 1600
●		0,5	10	250	590	0.18	1,15	0,3 - 1000
●		1						
●		2						
▽	CMC RK 5000	0,5	1,6	-	670	0		40 - 1600
▼		1	1,2					

Fig. 10 – *Dimensionless, standardized material function of viscoelastic fluids presented in Fig. 9*[4]

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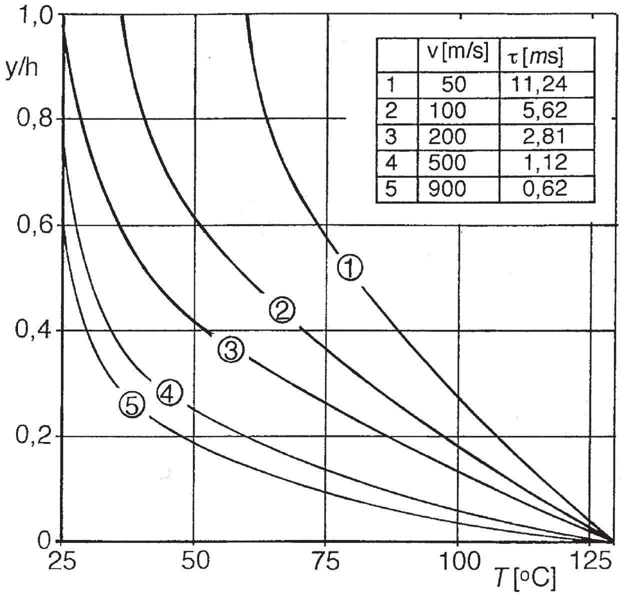
There is a wealth of materials whose behaviour depends on variable physical properties. Paper is one of them. In Fig. 11 the influence of the system temperature and of the duration of shear on the standardized paper coordinate is given. (h – paper thickness in the slot. At $y = 0$ the paper track is in contact with the heating drum, at $y = 1$ it sticks to the smoothing drum.) We learn from this plot that paper has a strong shear-thickening behaviour (dilatant viscosity) which also strongly depends on temperature.



	v [m/s]	τ [ms]
1	50	11,24
2	100	5,62
3	200	2,81
4	500	1,12
5	900	0,62

Fig. 11 – *The dependence of normalized paper thickness/temperature for paper strip under the following process conditions: heat roll temperature: 130 °C; linear pressure: 300 Nmm^{-2}; residence time in τ/ms in the slot between the rolls.*[5]

The following question is posed: Which paper quality may we use in model experiments to obtain reliable information about the behaviour of the actual paper strip in the smoothing process?

The answer is the same: We have to transform the dependence in Fig. 11 into the dimensionless, standardized material function, and then try to obtain a model paper pulp; the paper strip obtained from it meets the viscosity behavior of the original strip.

Literature

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